

Calculus & Analytical Geometry-I

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UPDATED VERSION

Exclusive thanks to Mahar Azahar (Lodhran)

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$
then f has relative At x_0

- ▶ Minima
- ▶ Maxima
- ▶ None of these

Question No: 2 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) < 0$
then f has relative At x_0

- ▶ Minima
- ▶ Maxima
- ▶ None of these

Question No: 3 (Marks: 1) - Please choose one

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \text{-----}$$

- ▶ 2
- ▶ 4

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- ▶ 1
- ▶ ∞

Question No: 4 (Marks: 1) - Please choose one

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \text{-----}$$

- ▶ 1
- ▶ 0
- ▶ e
- ▶ None of these

Question No: 5 (Marks: 1) - Please choose one

$$\frac{d(\tan x)}{dx} =$$

- ▶ $\sec x$
- ▶ $\sec^2 x$
- ▶ $\csc x$
- ▶ $\csc^2 x$

Question No: 6 (Marks: 1) - Please choose one

If $xy = 4$ then $\frac{dy}{dx} =$

- ▶ 0
- ▶ $-\frac{1}{x^2}$
- ▶ $\frac{4}{x^2}$
- ▶ $-\frac{4}{x^2}$

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Question No: 7 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$$\frac{d}{dx}((c) \{h(x)\}) = \underline{\hspace{2cm}}$$

- ▶ $\frac{d}{dx}(h(x))$
- ▶ $\frac{dx}{d}(h(cx))$
- ▶ $\frac{d}{dx}(h(cx))$
- ▶ $c \frac{d}{dx}(h(x))$

Question No: 8 (Marks: 1) - Please choose one

Suppose that f and g are differentiable functions of x then

$$\frac{d}{dx}[f][g] =$$

- ▶ $\frac{[f'] [g] - [f] [g']}{[f'] [g']^2}$
- ▶ $[f'] [g']^2$
- ▶ $[f'] [g] + [f] [g']$
- ▶ $[f'] [g] - [f] [g']$

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Question No: 9 (Marks: 1) - Please choose one

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

The power rule, holds if n is _____

- ▶ An integer
- ▶ A rational number
- ▶ An irrational number
- ▶ All of the above

Question No: 10 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denotes two distinct points in that interval. If $f(x_1) = f(x_2)$ for all points x_1 and x_2 then

which of the following statement is correct?

- ▶ f is a decreasing function
- ▶ f is an increasing function
- ▶ f is a constant function

Question No: 11 (Marks: 1) - Please choose one

If $f'(x) < 0$ on an open interval (a,b) then which of the following statement is correct?

- ▶ f is concave up on (a, b).
- ▶ f is concave down on (a, b)
- ▶ f is linear on (a, b).

Question No: 12 (Marks: 1) - Please choose one

$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

What does 'n' represent in Riemann Sum ?

- ▶ No. of Circles
- ▶ No. of Rectangles
- ▶ No. of Loops
- ▶ No. of Squares

Question No: 13 (Marks: 1) - Please choose one

If f is continuous function such that $\lim_{x \rightarrow -\infty} f(x) = +\infty$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$ then f has _____ on $(-\infty, +\infty)$

- ▶ maximum value but no minimum
- ▶ minimum value but no maximum
- ▶ both maximum and minimum value

Question No: 14 (Marks: 1) - Please choose one

$$\int_2^t \frac{x^2}{2} dx$$

The expression _____ , represents a function of :

- ▶ t
- ▶ x
- ▶ 2
- ▶ Both t and x

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Question No: 15 (Marks: 1) - Please choose one

$$\int cf(x)dx = \underline{\hspace{2cm}}$$

if c is a constant

- ▶ 0
- ▶ c
- ▶ $\int f(cx)dx$
- ▶ $c \int f(x)dx$

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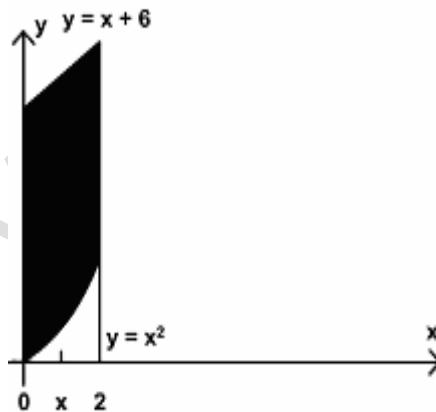
Question No: 16 (Marks: 1) - Please choose one

Sigma notation is represented by which of the following Greek letter?

- ▶ χ
- ▶ η
- ▶ Σ
- ▶ ψ

Question No: 17 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by :

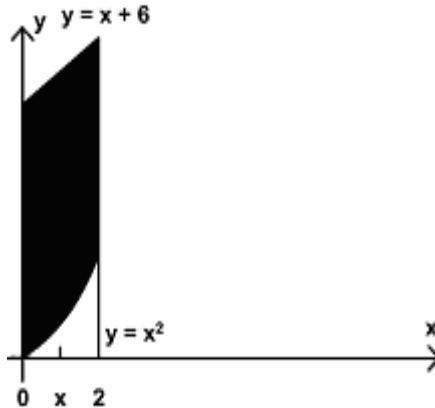


- ▶ $y = x + 6$
- ▶ $y = x^2$
- ▶ $x = 2$

► $x = 0$

Question No: 18 (Marks: 1) - Please choose one

In the following figure, the area bounded on the sides by the lines are :



► $x = 0$

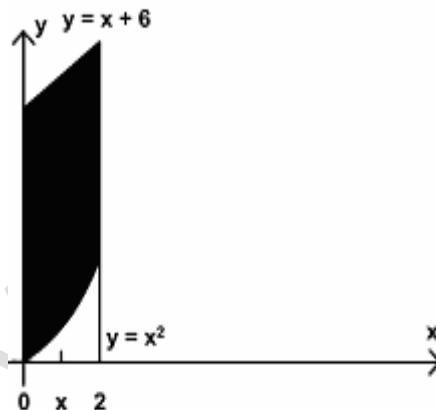
► $x = 2$

► $x = 0 \text{ and } x = 2$

► $x = 6$

Question No: 19 (Marks: 1) - Please choose one

What is the area of the region in the following figure?



►
$$A = \int_0^2 [(x+6) - (x^2)] dx$$

►
$$A = \int_x^2 [(x+6) - (x^2)] dx$$

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$$A = \int_0^2 [(x+6) + (x^2)] dx$$



$$A = \int_0^x [(x+6) - (x^2)] dx$$



Question No: 20 (Marks: 1) - Please choose one

$$y = f(x) = 3x + 1$$

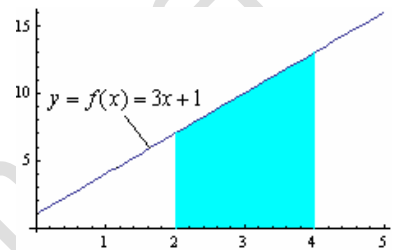
Which of the following is approximate area under the curve over the interval $[2, 4]$, evaluated by using the formula

$$Area = f(x_1^*) \Delta x + f(x_2^*) \Delta x$$

If the interval $[2, 4]$ is divided into two sub-intervals of equal

length and x_1^* and x_2^* are left endpoint of each sub-interval.

- 17
- **20**
- 23



Question No: 21 (Marks: 1) - Please choose one

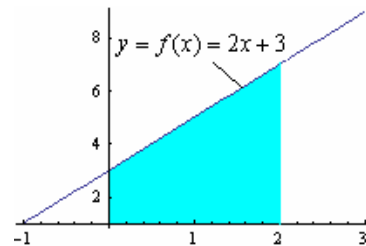
$$y = f(x) = 2x + 3$$

Which of the following is approximate area under the curve over the interval $[0, 2]$, evaluated by using the formula

$$Area = f(x_1^*) \Delta x + f(x_2^*) \Delta x$$

If the interval $[0, 2]$ is divided into two sub-intervals of equal length and x_1^* and x_2^* are right endpoint of each sub-interval.

- 8
- **10**
- 12



Question No: 22 (Marks: 1) - Please choose one

If $x > 0$ then $\frac{d}{dx}[\ln x] =$ _____

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- ▶ 1
- ▶ x
- ▶ $\frac{1}{x}$
- ▶ $\ln \frac{1}{x}$

Question No: 23 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on $[a,b]$ and c is a constant, then

$$\int_a^b c [f(x) + g(x)] dx = \underline{\hspace{2cm}}$$

- ▶ $\int_a^b f(cx)dx + \int_a^b g(cx)dx$

- ▶ $\int_a^b f(x) dx + \int_a^b g(x) dx$

- ▶ $c \int_a^b f(x)dx + c \int_a^b g(x)dx$

- ▶ 0

Question No: 24 (Marks: 1) - Please choose one

If the function f is continuous on $[a,b]$ and if $f(x) \geq 0$ for all x in $[a,b]$, then which of the following gives area under the curve $y = f(x)$ over the interval $[a,b]$?

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n [x_k][f(x_k)] \quad \text{where } n \text{ is number of subdivisions of } [a, b]$$

- ▶ $\int_a^b f(x) dx$

- ▶ $\pi[\text{radius}]^2$
- ▶ (Width) (Height)

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Question No: 25 (Marks: 1) - Please choose one

Let region R in the first quadrant enclosed between $y = 3x$ and $y = 2x^2$ is revolved about the x-axis. Which of the following equation gives the volume of a solid by cylindrical shells?

▶ $V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2)dx$

▶ $V = \int_0^{\frac{3}{2}} x(3x - 2x^2)dx$

▶ $V = \int_0^{\frac{3}{2}} 2\pi (3x - 2x^2)dx$

▶ $V = \int_{-1}^{\frac{3}{2}} 2\pi (3x - 2x^2)dx$

Question No: 26 (Marks: 1) - Please choose one

Let f is a smooth function on $[a, b]$. What will be the arc length L of the curve $y = f(x)$ from $x = a$ to $x = b$?

▶ $L = \int_b^a \sqrt{1 + [f'(x)]^2} dy$

▶ $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$

$$L = \int_0^a \sqrt{1 + [f'(x)]} dy$$



$$L = \int_a^b \sqrt{1 + [f'(x)]} dx$$



Question No: 27 (Marks: 1) - Please choose one

If f is continuous on (a, b] but does not have a limit from the right then the

$$\int_a^b f(x) dx = \lim_{l \rightarrow a} \int_l^b f(x) dx$$

integral defined by

is called :

- Improper
- Proper
- Line

Question No: 28 (Marks: 1) - Please choose one

$$\frac{a_{n+1}}{a_n} > 1$$

For a sequence $\{a_n\}$ if the ratio of successive terms then the sequence is known as:

- Increasing
- Decreasing
- Nondecreasing
- Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

$$\frac{a_{n+1}}{a_n} < 1$$

For a sequence $\{a_n\}$ if the ratio of successive terms then the sequence is known as:

- Increasing
- Decreasing
- Nondecreasing
- Nonincreasing

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Question No: 30 (Marks: 1) - Please choose one

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx$$

Consider the indefinite integral

Let $t = x^3 + 2x^2 + x - 3$

Is the following substitution correct?

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx = \int \frac{1}{t} dt$$

- ▶ Yes
- ▶ No

Question No: 31 (Marks: 1) - Please choose one

$$\rho = \lim_{k \rightarrow \infty} \frac{u_{k+1}}{u_k}$$

The series $\sum u_k$ be a series with positive terms and suppose that if $\rho = 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

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Question No: 32 (Marks: 1) - Please choose one

$$\sum u_k$$

The series be a series with positive terms and suppose that

$\rho = \lim_{k \rightarrow \infty} \sqrt[k]{u_k} = \lim_{k \rightarrow \infty} (u_k)^{\frac{1}{k}}$ if $\rho = 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 33 (Marks: 1) - Please choose one

$$\sum_{k=1}^{\infty} |u_k| = |u_1| + |u_2| + |u_3| + \dots + |u_k| + \dots$$

If the series $\sum_{k=1}^{\infty} u_k = u_1 + u_2 + u_2 + \dots + u_k + \dots$ converges, then which of

$$\sum_{k=1}^{\infty} u_k = u_1 + u_2 + u_2 + \dots + u_k + \dots$$

the following is true for ?

- ▶ Converges
- ▶ Diverges
- ▶ Gives no information

Question No: 34 (Marks: 1) - Please choose one

Let $\sum u_k$ be a series with nonzero terms and suppose that if $\rho = \lim_{k \rightarrow \infty} \frac{|u_{k+1}|}{|u_k|} = +\infty$, then which of the following is true?

▶ Then the series $\sum u_k$ diverges

- ▶ The series $\sum u_k$ converges absolutely and therefore converges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 35 (Marks: 1) - Please choose one

$$\int_{-1}^1 (x-1) dx =$$

- ▶ -2
- ▶ 0
- ▶ 2
- ▶ 4

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Question No: 36 (Marks: 1) - Please choose one

How many critical points exist for a function f if

$$f'(x) = (x - 3)(x - 2)$$

- ▶ Zero
- ▶ One
- ▶ Two
- ▶ Four

Question No: 37 (Marks: 1) - Please choose one

$$\log_b ac = \underline{\hspace{2cm}}$$

- ▶ $\log_b a + \log_b c$
- ▶ $\log_b a - \log_b c$
- ▶ $\frac{\log_b a}{\log_b c}$
- ▶ $(\log_b a)(\log_b c)$

Question No: 38 (Marks: 1) - Please choose one

$$\log_b a^r = \underline{\hspace{2cm}}$$

- ▶ $a \log_b r$
- ▶ $r \log_b a$
- ▶ $\frac{\log_b a}{\log_b r}$
- ▶ $\log_b a + \log_b r$

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Question No: 39 (Marks: 1) - Please choose one

$$y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} ; 0 \leq x \leq 2$$

Let

then which of the following is the length of the curve?

$$L = \int_0^2 \sqrt{1 + \left| \frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right|^2} dx$$



$$L = \int \sqrt{1 + \left| \frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right|^2} dx$$



$$L = \int_0^2 \sqrt{1 + \left| \frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right|^2} dx$$



$$L = \int_0^2 \sqrt{1 + \left| \frac{d}{dx} \left(\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right|^2} dx$$



Question No: 40 (Marks: 1) - Please choose one

Which of the following are *first two* terms for the Taylor series of $f(x) = e^{-x}$ at $x = 0$?

▶ $1 + (1)(x - 0)$



▶ $1 + (-1)(x + 0)$



▶ $1 + (-1)(x - 0)$



▶ $(-1)(x - 0)$



Question No: 41 (Marks: 2)

3

$$\int_2^3 (1-x) dx$$

Evaluate the integral

$$\begin{aligned}
 & \int_2^3 (1-x) dx \\
 &= \left[x - x^2 \frac{1}{2} \right]_2^3 \\
 &= \frac{1}{2} [2x - x^2]_2^3 \\
 &= \frac{1}{2} (2(3-2) - (3-2)^2) \\
 &= \frac{1}{2} (2-1) \\
 &= \frac{1}{2}
 \end{aligned}$$

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Question No: 42 (Marks: 2)

$$\int_2^{+\infty} \frac{dx}{x^2}$$

Evaluate the improper integral

Question No: 43 (Marks: 2)

A function $f(x) = x^2 - 4x - 9$ has critical point 2 in an interval $[0, 5]$. Find the maximum value of the function and point having this value.

Question No: 44 (Marks: 3)

$$\int \frac{5 - 6 \sin^2 x}{\sin^2 x} dx$$

Evaluate:

$$\int \frac{5 - 6 \sin^2 x}{\sin^2 x} dx$$

Question No: 45 (Marks: 3)

Find the area of the region bounded by the curve $y = x^2$, $x > 0$, and bounded on the sides by the lines $y = 1$ and $y = 4$

$$y = x^2, \quad x > 0$$

So we have

$$\begin{aligned}
 A &= \int_1^4 x^2 dx \\
 &= \left| \frac{x^3}{3} \right|_1^4 \\
 &= \frac{1}{3} (4-1)^3 \\
 &= \frac{1}{3} (3)^3 \\
 &= 9
 \end{aligned}$$

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Question No: 46 (Marks: 3)

Determine whether the following sequence converges or diverges. If it converges, find the limit.

$$\lim_{n \rightarrow \infty} \frac{5n^2 - 1}{20n + 7n^2}$$

Question No: 47 (Marks: 5)

Use the Alternating series Test to determine whether the given series converges

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot n!}{2^n}$$

Question No: 48 (Marks: 5)

Evaluate the integral

$$\int_{\frac{\pi}{2}}^0 \frac{1 + \cos 2t}{2} dt$$

Solution

$$\int_{\frac{\pi}{2}}^0 \frac{1 + \cos 2t}{2} dt$$

$$u = 2t$$

$$du = 2dt$$

$$dt = \frac{du}{2}$$

$$\text{so}$$

$$= \frac{1}{4} \int_{\frac{\pi}{2}}^0 (1 + \cos u) du$$

$$= \frac{1}{4} \left[u + \sin u \right]_{\frac{\pi}{2}}^0$$

$$= \frac{1}{4} \left[2t + \sin 2t \right]_{\frac{\pi}{2}}^0$$

$$= \frac{1}{4} \left(2 \cdot \frac{\pi}{2} + \sin 2 \cdot \frac{\pi}{2} \right)$$

$$= \frac{1}{4} (\pi + \sin \pi)$$

$$= \frac{1}{4} (\pi + 0)$$

$$= \frac{\pi}{4}$$

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Question No: 49 (Marks: 5)

Evaluate the sums

$$\sum_{k=1}^5 k(3k + 5)$$

$$= 1(3 + 5) + 2(6 + 5) + 3(9 + 5) + 4(12 + 5) + 5(15 + 5)$$

$$= 8 + 22 + 3(45) + 4(60) + 5(75)$$

$$= 8 + 22 + 135 + 240 + 375$$

$$= 780$$

Question No: 50 (Marks: 10)

Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x – axis.

$$y = 1 + x^3, \quad x = 1, x = 2, y = 0$$

$$\begin{aligned} \text{from } V &= \int_a^b \pi [f(x)]^2 dx \\ V &= \int_1^2 \pi [1 + x^3]^2 dx \\ V &= \int_1^2 \pi [1 + x^5 + 2x^3] dx \\ V &= \pi \int_1^2 (1 + x^5 + 2x^3) dx \\ V &= \pi \left[x + \frac{1}{6}x^6 + \frac{1}{2}x^4 \right]_1^2 \\ V &= \pi \left((2-1) + \frac{1}{6}(2-1)^6 + \frac{1}{2}(2-1)^4 \right) \\ V &= \pi \left\{ (2-1) + \frac{1}{6}(2-1)^6 + \frac{1}{2}(2-1)^4 \right\} \\ V &= \pi \left((1) + \frac{1}{6} + \frac{1}{2} \right) \\ V &= \frac{\pi(6+1+3)}{6} \\ V &= \frac{\pi(10)}{6} = \pi \frac{5}{3} \end{aligned}$$

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This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at gulshanvu@yahoo.com, with reference to the concerned paper's number.