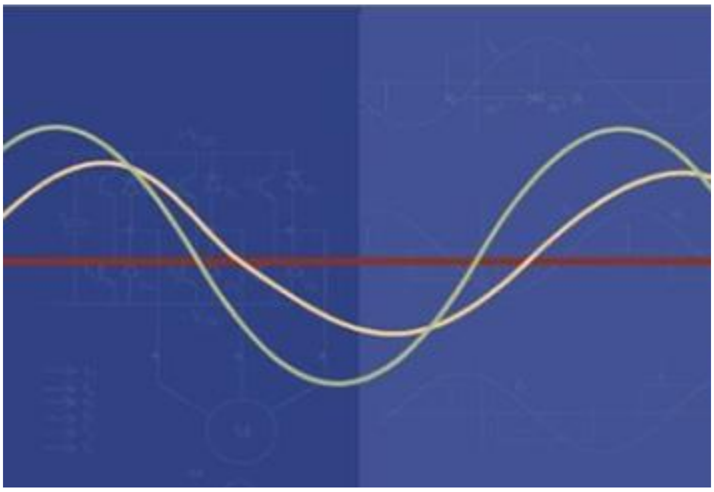
UNESCO-NIGERIA TECHNICAL &
VOCATIONAL EDUCATION
REVITALISATION PROJECT-PHASE II

NATIONAL DIPLOMA IN ELECTRICAL ENGINEERING TECHNOLOGY



ELECTRICAL CIRCUITS (I)

COURSE CODE: EEC 239

YEAR II- SEMESTER III

THEORY

Version 1: December 2008

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At the end of this week, the students are expected to:

- ♦ State different mathematical forms of representing a.c signals
- ♦ Convert a.c signal in polar form to the j-notation
- ♦ Subtract, add, multiply and divide phasor using j-operator
- ♦ Solve simple problems using j-notation

1.1 MATHEMATICAL FORMS OF REPRESENTING A.C SIGNALS

Generally, A.C signal may be represented in the following mathematical form;

a) Trigonometric form, $Z = r(\text{Cos}\ \theta + \sin\theta)$ (1.1)

b) Polar form, $Z = r\angle\theta$ (1.2)

c) J – notation form, $Z = x+jy$ (1.3)

1.2 CONVERSION OF A.C SIGNAL IN POLAR FORM TO THE j – NOTATION FORM

Example 1.1**:** Express $18\angle -56.3$ in j – notation form

Solution:

Step I: Make a sketch as shown on fig 1.1 to measure $\theta = -56.3^0$ with respect to the real axis and $r = OA = 18$

Step II: Find x using trigonometric ratio,

$\text{Cos}\theta = x/18$

i.e $x = 18\cos\theta = 18\ \text{Cos}\ (-56.3^0)$

$\therefore\ x \approx 10$

Step III: Find y using trigonometric ratio

$\text{Sin}\ \theta = Y/18$

i.e $y = 18\sin\theta = \sin(-56.3^O)$

$\therefore\ y \approx -15$

Step IV: finally obtain the required result as $x + jy = 10 - j15$

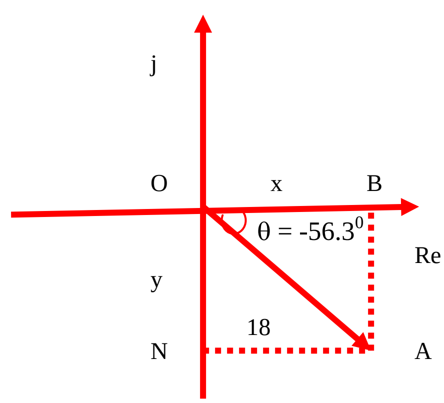


Fig 1.1

Example 1.2: Express $5\angle 53.1^o$ in j – notation form

Solution:

Step I: Make a sketch as shown on fig 1.2 to measure $\theta = 53.1^0$ with respect to the real axis and $r = OA = 5$

Step II: Find x using trigonometric ratio

$\mathrm{Cos}\theta = x/5$

i.e $x = 5\cos\theta = 5\,\mathrm{Cos}\,(53.1^0)$

$\therefore\ x \approx 3$

Step III: Find y using trigonometric ratio

$\mathrm{Sin}\ \theta = Y/5$

i.e $y = 5\sin\theta = \sin(53.1^O)$

$\therefore\ y \approx 4$

Step IV: finally obtain the required result as $x + jy = 3 - j4$

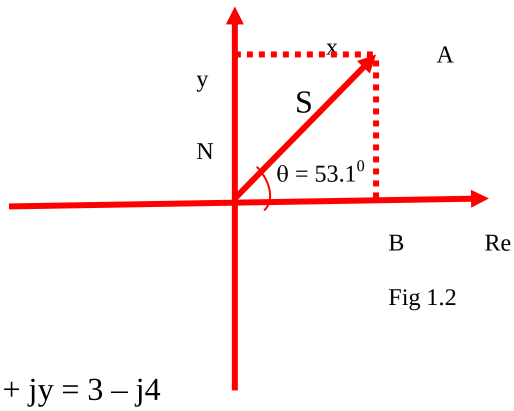


Fig 1.2

1.3 SUBTRACTION, ADDITION, MULTIPLICATION AND DIVISION OF PHASOR USING j- OPERATOR.

1.3.1 Addition/Subtraction of Phasor

The most convenient way to add or subtract phasor is to first convert them to j – notation form, if they are in polar form. Once the phasor are in j – notation form, the real (x) components and the imaginary (y) components can be algebraically added or subtracted, as may be required. The answer (the sum or difference) can be left in j – notation form or it can be converted back to polar form if desired.

If two phasor are Z_1 and a +jb and Z_2 = c+jd, then addition of phasor gives: $Z_1 + Z_2 = (a+jb) + (c+jd) = (a+c) + j(b+d)$

Subtraction of phasor gives: $Z_1 - Z_2 = (a-c) + j(b-d)$

Example 1.3: If $V_1 = -10 + j20$ and $V_2 = 20 + j30$, find the sum of V_1 and V_2, express the result in polar form.

Solution

$V_1 = -10+j20$

$V_2 = 20 + j30$

$V_3 = V_1 + V_2 = -10+ 20+j20 + j30 = 10 +j50$

Finally, $V_3 = \sqrt{10^2+50^2}\ \mathrm{Tan}^{-1}\left({}^{50}/_{10}\right) = 51\angle 78.7^0$

Example 1.4 Subtract $I_1 = 3\angle{-56.3^0}$ from $I_2 = 5.8\angle 30^0$

Solution

Step I: Convert first to j – notation form to get (using the trigonometric form)

$I_1 = 3\{\cos(-56.3^0) + j\sin(-56.3^0)\}$

$= 3\{0.5548 + j(-0.8320)\} = 1.66 - j2.50$

similarly, $I_2 = 5.8(\cos 30^0 + j\sin 30^0) \approx 5.02 + j2.90$

Step II: $I_2 = 5.02+j2.90$

$I_1 = 1.66-j2.50$

Subtracting I_1 from I_2, we get

$I_3 = I_2 - I_1 = 3.36 + j5.40$

1.3.2 Multiplication/Division of Phasors

The most convenient way to multiply or divide phasors is to first convert them to polar form, if they are in j – notation form. Once the phasor are in polar form, to multiply polar phasor, just multiply the magnitude and algebraically add the phase angle. To divide phasor, the magnitudes are divided and the angles are algebraically subtracted.

If $Z_1 = r_1 \angle\theta_1$ and $Z_2 = r_2 \angle\theta_2$, then multiplying Z_1 by Z_2 we get $Z_1 Z_2 = r_1 r_2 \angle(\theta_1+\theta_2)$

If $Z_1 = r_1 \angle\theta_1$ and $Z_2 = r_2 \angle\theta_2$, then dividing Z_1 by Z_2 we get $Z_1/Z_2 = r_1/r_2 \angle(\theta_1-\theta_2)$

1.4 SOLVED SIMPLE PROBLEMS USING J-NOTATION

Example 1.5: Given the following two vectors $A = 20\angle 60^0$ and $B = 5\angle 30^0$ perform the following indicated operation (i) A x B (ii) A/B

Solution:

i. $A \times B = 20\angle 60^0 \times 5\angle 30^0 = 20 \times 5\angle 60^0 + 30^0 = 100\angle 90^0$
ii. $A/B = 20\angle 60^0/5\angle 30^0 = 4\angle 60^0\text{-}30^0 = 4\angle 30^0$

Example 1.6: perform the following operation and the final result may be given in polar for; (8 +j6)x(-10-j7.5)

Solution

$(8+j6)x(-10-j7.5)= -80-j60-j60-j^{2}45=-80+45-j120$

$= -35-j120 = \sqrt{(-35)^2+ (-120)^2} \angle \tan^{-1}(^{120}/_{35}) = 125\angle 73.7^0$

At the end of this week, the students are expected to:

- ♦ Draw to scale phasor diagram for a.c circuits
- ♦ Show with the aid of waveforms diagrams that the current in a capacitor circuit leads voltage and the current in the inductive circuit lags the voltage.
- ♦ Distinguish between inductive and capacitive reactances.
- ♦ Draw voltage and current waveforms on same axis to show lagging and leading angles.

1.5 PHASOR DIAGRAMS FOR A.C CIRCUITS DRAWN TO SCALE

For a pure inductive circuit, the voltage across the inductor (V_L) leads the currents (I) flowing through it by 90^0. Taking a scale of 1cm:2V, and 1cm:3A, the Phasor diagram is drawn to scale as shown in figure1.3 (b)

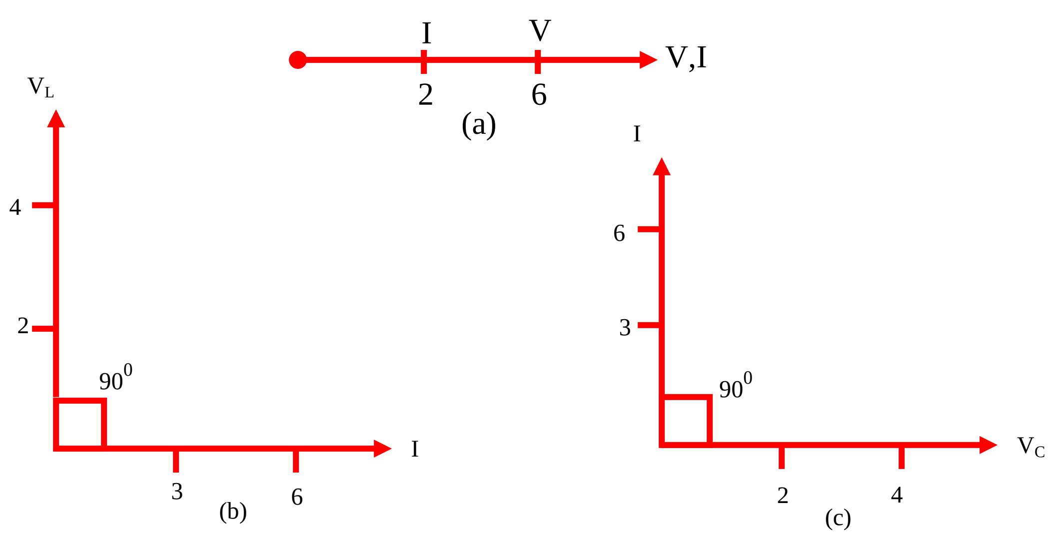


Fig 1.3 Phasor diagram for A.C circuit drawn to scale:

(a) Resistive circuit

(b) Inductive circuit

(c) Capacitive circuit

For a pure capacitive circuit, the current (I) flowing through the capacitor leads the voltage across it (Vc) by 90^0. Taking a scale of 1cm:3A, and 1cm:2V, the phasor diagram is drawn to scale as shown in fig 1.3 (c)

For a pure resistive circuit, the voltage across the resistor (V) is in phase with the current (I) flowing through it. Taking a scale of 1cm:3A, and 1cm:2V, the Phasor diagram is drawn to scale as shown in figure 1.3 (a)

1.6 DERIVATIONS WITH THE AID OF WAVEFORMS DIAGRAMS THAT THE CURRENT IN A CAPACITIVE CIRCUIT LEADS VOLTAGE AND THE CURRENT IN THE INDUCTIVE CIRCUIT LAGS THE VOLTAGE

1.6.1 Current And Voltage In An Inductive Circuit

Suppose the instantaneous value of the current through a coil having inductance L henrys and negligible resistance to be represented by

$$i = I_m \sin\omega t = I_m \sin 2\pi ft \qquad (1.4)$$

where t is the time, in seconds, after the current has passed through zero from negative to positive values, as shown in fig 1.4.

Suppose the current to increase by di ampere in dt seconds, then instantaneous value of induced e.m.f is

$$e = L\frac{di}{dt}$$

$$= LI_m \frac{d}{dt}(\sin 2\pi ft)$$

$$= 2\pi fL\, I_m \cos 2\pi ft = V_m \cos 2\pi ft, \text{ where } V_m = 2\pi fLI_m$$

$$e = V_m \sin(2\pi fL + {}^{\pi}/_{2}) \qquad (1.5)$$

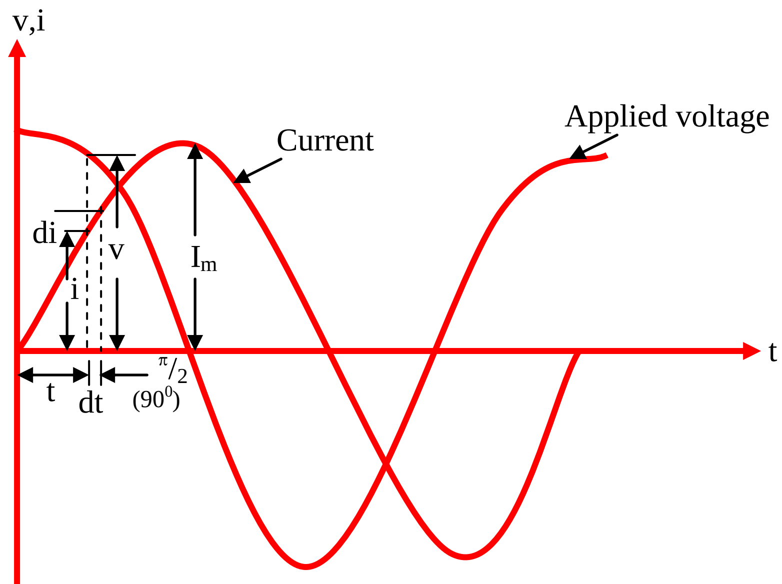


Fig 1.4: voltage and current waveforms for a purely inductive circuit

The induced e.m.f is represented by the curve in fig 1.4, leading the current by a quarter of a cycle.

Since the resistance of the circuit is assumed negligible, the whole of the applied voltage is equal to the induced e.m.f, therefore instantaneous value of applied voltage is

$V = e$

$V = V_m \sin(2\pi ft + {}^{\pi}/_{2})$ (1.6)

Comparism of expressions (1.4) and (1.6) shows that the applied voltage leads the current by a quarter of a cycle.

1.6.2 Current and voltage in a capacitive circuit

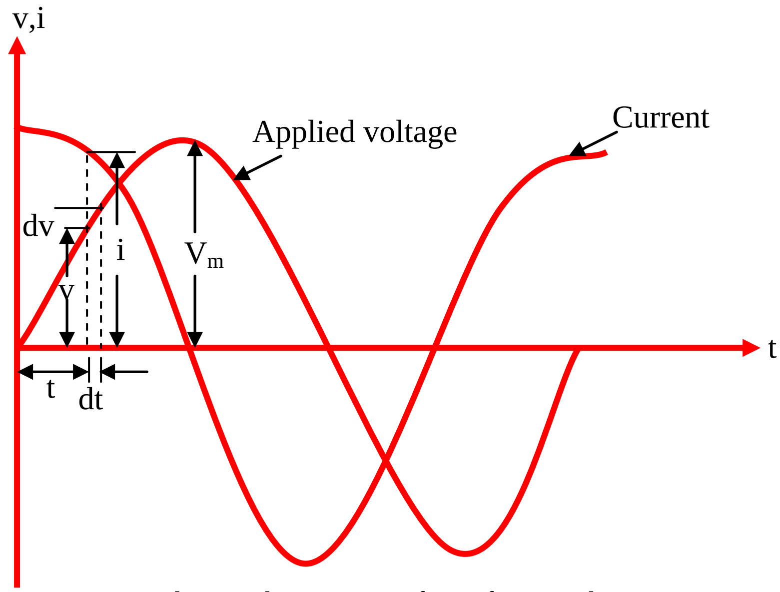


Fig 1.5: voltage and current waveforms for a purely capacitive circuit

Suppose that the instantaneous value of the voltage applied to a capacitor having capacitance C farad is represented by

$V = V_m \sin\omega t = V_m \sin 2\pi ft$ ` (1.7)

If the applied voltage increases by dv volts in dt seconds (fig,1.5) then, instantaneous value of current is

$i = C\, {}^{dv}/_{dt}$

$= C\, {}^{d}/_{dt}(V_m \sin 2\pi ft)$

$= 2\pi fCV_m \cos 2\pi ft = {}^{V_m}/_{X_C} \cos 2\pi ft$

$i = I_m \sin(2\pi ft + {}^{\pi}/_{2})$ (1.8)

where $I_m = 2\pi fCV_m$

Comparism of expression (1.7) and (1.8) shows that the current leads the applied voltage by a quarter of a cycle.

1.7 DISTINGUISH BETWEEN INDUCTIVE AND CAPACITIVE REACTANCES

1.7.1 Inductive Reactance

From the expression $V_m = 2\pi fLI_m$, $V_m/I_m = 2\pi fL$

If I and V are the r.m.s values, then

$$\frac{V}{I} = \frac{0.707\,V_m}{0.707 I_m} = 2\pi f L$$ inductive reactance

The inductive reactance is expressed in ohms and is represented by the symbol X_L.

The inductive reactance is proportional to the frequency and the current produced by a given voltage is inversely proportional to the frequency, as shown in fig 1.6

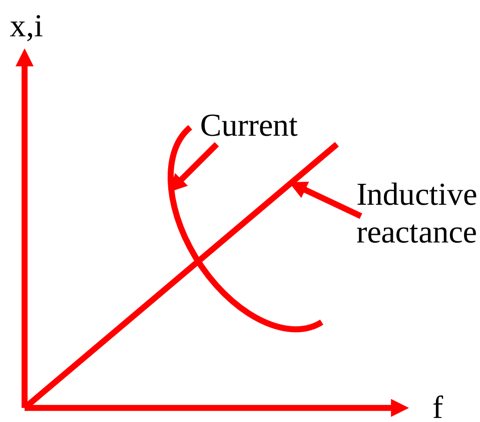


Fig 1.6: variation of reactance and current with frequency for a purely inductive circuit

1.7.1 Capacitive reactance

From the expression $-I_m = 2\pi fCV_m$

$$V_m/I_m = \frac{1}{2\pi fC} = \text{capacitive reactance} \qquad (1.10)$$

The capacitive reactance is expressed in ohms and is represented by the symbol X_c.

The capacitive reactance is inversely proportional to the frequency and the current produced by a given voltage is proportional to the frequency, as shown in fig 1.7

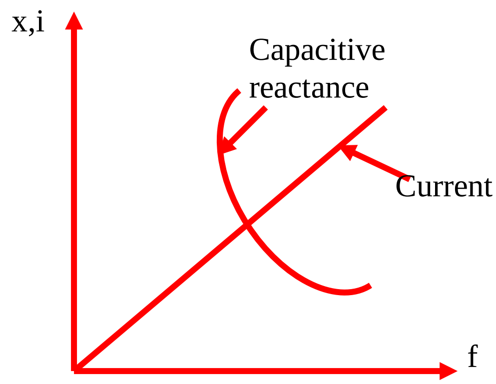


Fig 1.7: variation of reactance and current with frequency for a purely capacitive circuit

1.8 VOLTAGE AND CURRENT WAVEFORMS ON SAME AXIS SHOWING LAGGING AND LEADING ANGLES

The waveforms of voltage and current on same axis, showing leading and lagging angle (ϕ) is shown in fig 1.8

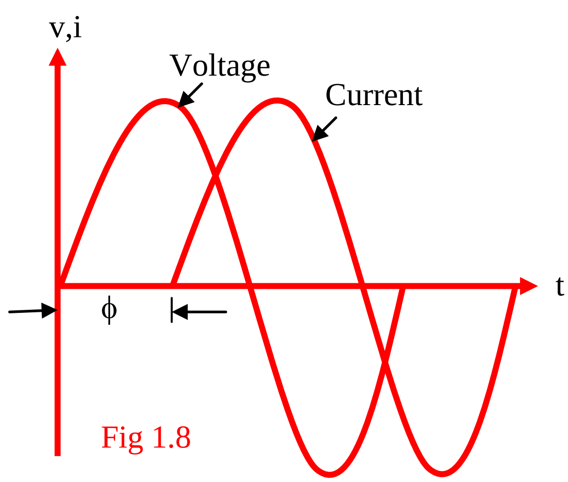


Fig 1.8

At the end of this week, the students are expected to:

- Draw phasor diagram for series and parallel a.c circuits
- Calculate voltage, current, power and power factor in series and parallel circuits
- Explain series and parallel resonance
- State conditions for series and parallel resonance

1.9 PHASOR DIAGRAMS FOR SERIES AND PARALLEL A.C CIRCIUT

1.9.1 Phasor diagrams for series a.c circuits:

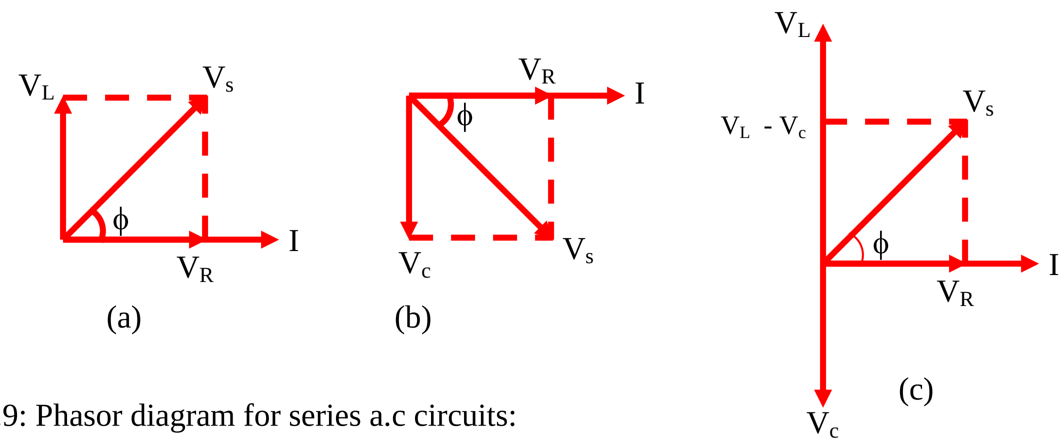


fig 1.9: Phasor diagram for series a.c circuits:

(a) R-L series circuit (b) R-C series circuit (c) R-L-C series circuit

1.9.2 Phasor diagram for parallel a.c circuits:

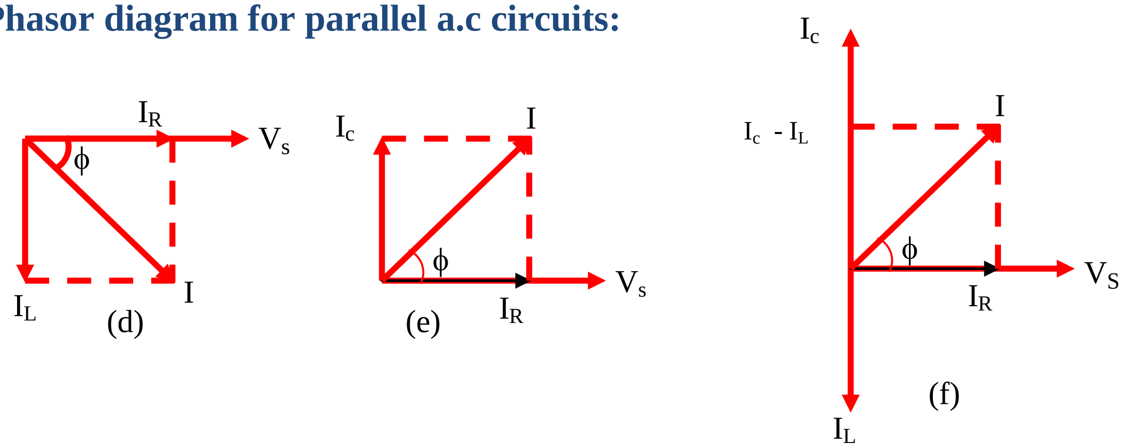


Fig 1.10: Phasor diagrams for parallel a.c circuits:

(d) R-L parallel circuit (e) RC parallel circuit (f) RLC parallel circuit

1.10 VOLTAGE, CURRENT, POWER AND POWER FACTOR CALCULATIONS IN SERIES AND PARALLEL CIRCUIT

Example 1.7

A resistance of 10Ω is connected in series with a pure inductance of `100mH and the circuit is connected across a 100V, 50Hz supply. Calculate (a) the circuit current (b) the voltage across each element (c) the power factor of the circuit (d) the power consumed.

Solution

(a) circuit current, $I = \frac{V_s}{Z} = \frac{100}{\sqrt{(10^2 + X_L^2)}}$.

And $X_L = 2\pi fL = 2\pi \times 50 \times 100 \times 10^{-3} = 31.42\Omega$

$$I = \frac{100}{\sqrt{(10^2 + 31.42^2)}} = 3.03A$$

(b) Voltage across the resistor, $V_R = IR = 3.03 \times 10 = 30.3V$

Voltage across the inductor, $V_L = IX_L = 3.03 \times 31.42 = 95.2V$

(c) From the phasor diagram of RL series circuit,

$\phi = \tan^{-1}(V_L/V_R) = \tan^{-1}(95.2/30.3) = 72.3^0$

$\therefore$ Power factor $= \cos\phi = \cos 72.3^0 = 0.3040$

(d) $P = I^2R = 3.03^2 \times 10 = 91.81W$

Example 1.8: A circuit having a resistance of 12Ω, an inductance of 0.15H and a capacitance of 100μF in series, is connected across a 100V, 50Hz supply. Calculate (a) the impedance (b) the current (c) the voltages across R,L and C (d) the power factor of the circuit

Solution

(a) $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$

$$= \sqrt{(12)^2 + \left(2x3.142x50x0.15 - \frac{1}{2x3.142x50x100x10^{-6}}\right)^2} = 19.4\Omega$$

(b) $I = V/Z = 100/19.4 = 5.15A$

(c) Voltage across R $= V_R = 12 \times 5.15 = 61.8V$

Voltage across L $= V_L = 2\pi \times 50 \times 0.15 \times 5.15 = 242.5V$

and voltage across C $= V_C = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} \times 5.15 = 164V$

(d) power factor = $\cos\phi$

Where $\phi = \cos^{-1}(V_R/V_S) = \cos^{-1}(61.8/100) = 51.8^0$

P.f = cos51.8 = 0.6184

Example 1.9: A circuit consists of a 115Ω resistor in parallel with a 41.5µF capacitor and is connected to a 230V, 50Hz supply (fig 1.11). Calculate: (a) the branch currents and the supply current (b) the power factor (c) the power consumed

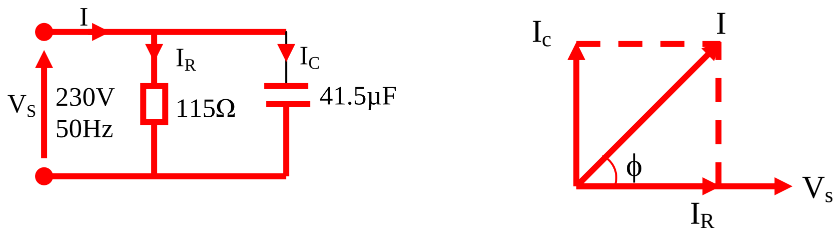


Fig 1.11: circuit and phasor diagrams for example 1.9

Solution

(a) $I_R = V_S/R = 230/115 = 2A$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi x50x41.5x10^{-6}} = 76.7\Omega$$

$I_C = V_S/X_C = 230/76.7 = 3A$

$I = \sqrt{(I_R^2 + I_L^2)} = \sqrt{(2^2 + 3^2)} = 3.6A$

(b) P.f = $\cos\phi$

and $\phi = \cos^{-1}(I_R/I) = \cos^{-1}(2/3.6) = 56.3^0$

$\therefore$ P.f = $\cos 56.3^0 = 0.5548$

(c) $P = I^2R = 3.6^2 \text{ x } 115 = 1490.4W$

Example 1.10 Three branches, possessing a resistance of 50Ω, an inductance of 0.15H and a capacitance of 100µF respectively, are connected in parallel across a 100V, 50Hz supply. Calculate: (a) the current in each branch (b) the supply current (c) the phase angle between the supply current and the supply voltage (d) the power factor of the circuit.

Solution

The circuit diagram for example 1.10 is shown in fig 1.12

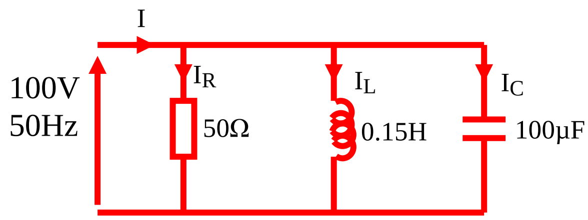


Fig 1.12: circuit diagram for example 1.10

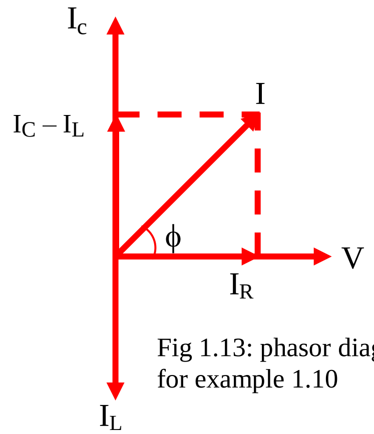


Fig 1.13: phasor diagram for example 1.10

(a) $I_R = 100/50 = 2A$

$I_L = \dfrac{100}{2 \times 3.142 \times 50 \times 0.15} = 2.12A$

and $I_C = 2 \times 3.14 \times 50 \times 100 \times 10^{-6} \times 100 = 3.14A$

(b) The resultant of I_C and I_L is

$I_C - I_L = 3.14 - 2.12 = 1.02A$

$I = \sqrt{[I_R^2 + (I_C - I_L)^2]} = \sqrt{(2^2 + 1.02^2)} = 2.24A$

(c) From fig 1.13:

$\cos\phi = I_R/I = 2/2.24 = 0.893$

$\phi = \cos^{-1}(0.893) = 26.7^0$

(d) $P.f = \cos\phi = \cos 26.7 = 0.8934$

1.11 SERIES AND PARALLEL RESONANCE

Generally, an ac circuit is said to be in resonance when the applied voltage V (with constant magnitude, but of varying frequency) and the resulting current I are in phase

1.11.1 Series resonance

If at some frequency of the applied voltage, $X_L = X_C$ the current in the

circuit is given by $I = \dfrac{V_S}{Z} = \dfrac{V_S}{\sqrt{[R^2 + (X_L - X_C)^2]}} = \dfrac{V_S}{\sqrt{R^2}} = \dfrac{V_S}{R}$

This condition where by $X_L = X_C$ in a series R-L-C circuit is called series resonance, and the frequency at which it occurs is called resonant frequency, f_0. The phasor diagram of the R-L-C series circuit at resonance is shown in fig 1.14

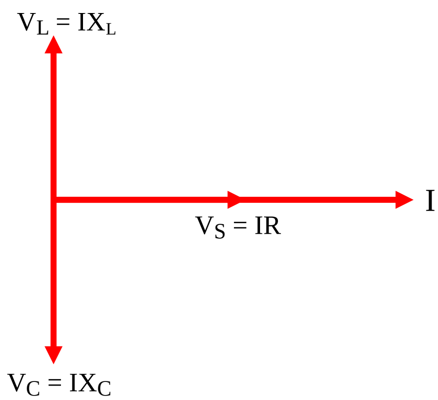
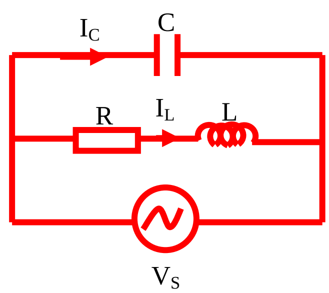


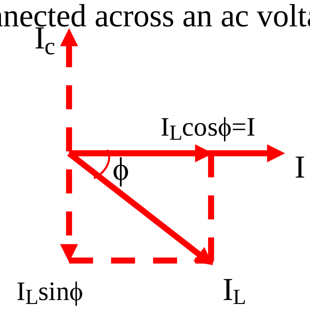
Fig 1.14: phasor diagram for an RLC series circuit at resonance

1.11.2 Parallel Resonance

Parallel resonance occurs when the active component of the current is in phase with the supply current. Figure 1.15 shows a parallel resonance circuit where a coil (RL) is connected in parallel with a capacitor C. This combination is connected across an ac voltage source



(a) Parallel circuit



(b) Phasor diagram

Fig 1.15: parallel resonance

1.12 CONDITIONS FOR SERIES AND PARALLEL RESONANCE

1.12.1 Conditions for series resonance

(i) The applied voltage V_S and the resulting current I are in phase

(ii) The net reactance is zero because $X_L = X_C$

(iii) The impedance Z of the circuit is minimum

(iv) The current in the circuit is maximum

(v) The resonant frequency is given by $fr = \frac{1}{2\pi\sqrt{(LC)}}$.

1.12.2 Condition for parallel resonance

(i) The applied voltage V_S and the resulting current Ir are in phase

(ii) The power factor is unity

(iii) The impedance of the current at resonance is maximum.

(iv) The value of current at resonance is minimum.

(v) The resonance frequency is given by

$$fr = \frac{1}{2\pi}\left(\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\right)$$

At the end of this week, the students are expected to:

- ♦ Prove the relevant formulae for Q-factor, dynamic impedance and bandwidth at resonance frequency
- ♦ Sketch current and impedance against frequency for series and parallel circuits
- ♦ Calculate the Q-factor for a coil; loss factor for a capacitor
- ♦ Explain with the aid of a diagram, bandwidth
- ♦ Solve problems involving bandwidth and Q-factor

1.13 DERIVATIONS OF Q-FACTOR, DYNAMIC IMPEDENCE AND BANDWIDTH AT RESONANCE FREQUENCY

1.13.1 Q-Factor of a Series Resonance

$$\text{Q-factor} = \frac{\text{Potential drop across the inductance at resonance}}{\text{Potential drop across the resistance at resonance}}$$

$$= \frac{IX_L}{IR} = X_L/R = \frac{2\pi f_0 L}{R} \qquad (1.11)$$

Also,

$$\text{Q-factor} = \frac{\text{Potential drop across the capacitance at resonance}}{\text{Potential drop across the resistance at resonance}}$$

$$= \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{2\pi f_0 CR} \qquad (1.12)$$

1.13.2 Q-factor of a Parallel Resonance

$$\text{Q-factor} = \frac{\text{Circulating current}}{\text{Supply current}} = \frac{I_L \sin\phi}{I} \qquad \text{(see fig 1.15b)}$$

$$= \tan\phi_L = \frac{X_L}{R} = \frac{2\pi f_0 L}{R} \qquad (1.13)$$

1.13.3 Dynamic Impedance of a Series Resonance

At series resonance, $X_L = X_C$ and the impedance in the circuit is

$$Z = \frac{V_S}{I} = Z_D$$

where Z_D is known as dynamic impedance

$$\therefore Z_D = \frac{V_S}{I} = \frac{V_S}{V_S/\sqrt{[R^2 + (X_L - X_C)^2]}} = \frac{V_S}{V_S/\sqrt{R^2}} = \frac{V_S\sqrt{R^2}}{V_S} = R$$

$$\therefore Z_D = R \qquad (1.14)$$

1.13.4 Dynamic Impedance of a Parallel Resonance

Consider the phasor diagram of fig 1.15(b)

$I = I_1 \cos\phi_1$, where $I_1 = \frac{V_S}{Z_1}$ and $\cos\phi_1 = \frac{R}{Z_1}$

$$\therefore\ I = \frac{V_S}{Z_1} \cdot \frac{R}{Z_1} = \frac{V_S R}{Z_1^2} \qquad (1.15)$$

Also, $I_C = I_1 \sin\phi_1$, where $\sin\phi_1 = \frac{X_L}{Z_1}$, and $I_C = \frac{V_S}{X_C}$

$\therefore \frac{V_S}{X_c} = \frac{V_S}{Z_1} \cdot \frac{X_L}{Z_1}$ or $Z_1^2 = X_L X_C$ where $X_L = \omega L$, $X_C = 1/\omega C$

$$\therefore\ Z_1^2 = L/C \qquad (1.16)$$

Putting eqtn (1.16) in (1.15) gives

$I = \frac{V_S RC}{L} \Rightarrow \frac{V_S}{I} = Z = \frac{L}{RC}$ = dynamic impedance

$$\therefore Z = L/RC \qquad (1.17)$$

1.13.5 Bandwidth at Resonance Frequency

The quality factor Q_0 (at resonance) can be expressed as the ratio of the resonant frequency to bandwidth (BW). Hence,

$Q_0 = \frac{f_0}{BW}$, $\Rightarrow BW = \frac{f_0}{Q_0}$.

1.14 SKETCH OF I AND Z AGAINST F FOR SERIES AND PARALLEL CIRCUIT

1.14.1 Sketch of Current (I) and Impedance (Z) against frequency (F) for Series Circuit

The sketch of I and Z against F for series circuit is shown in fig 1.16

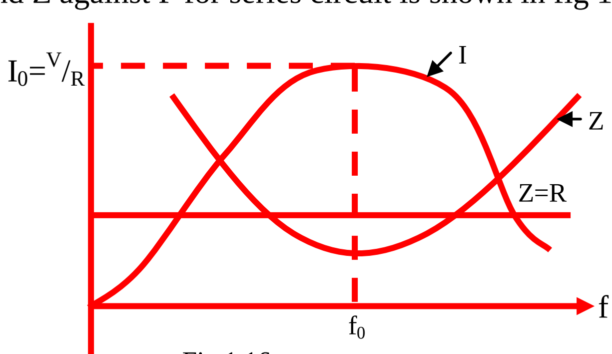


Fig 1.16

1.14.2 Sketch of current (I) and impedance (Z) against frequency (F) for parallel circuit

The sketch of I and Z against f for parallel circuit is shown in fig 1.17

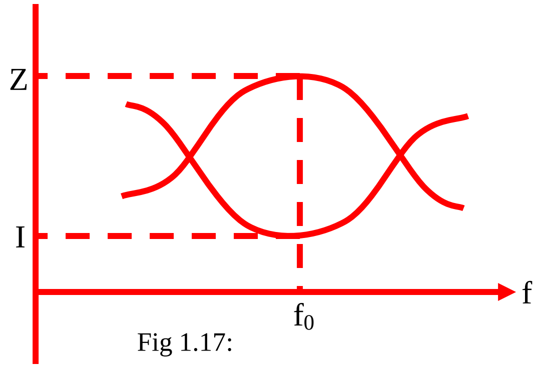


Fig 1.17:

1.15 CALCULATION OF Q-FACTOR FOR A COIL AND LOSS FACTOR FOR A CAPACITOR

Example 1.11 Determine the Q-factor of a coil whose resistance and inductance are 10Ω and 20mH, respectively, if the coil is at resonance at 1.2KHz.

Solution

$$Q_f = \frac{2\pi f_0 L}{R} = \frac{2\pi \text{ x } 1.2\ 10^3 \text{ x } 20 \text{ x } 10^{-3}}{10} = 15.1$$

Example 1.12 A capacitor has a capacitance of 10μF and an actual phase angle of 80^0. It is connected across a 200V, 50Hz line; find the loss factor of the capacitor

Solution

The circuit and phasor diagrams are shown in fig 1.18

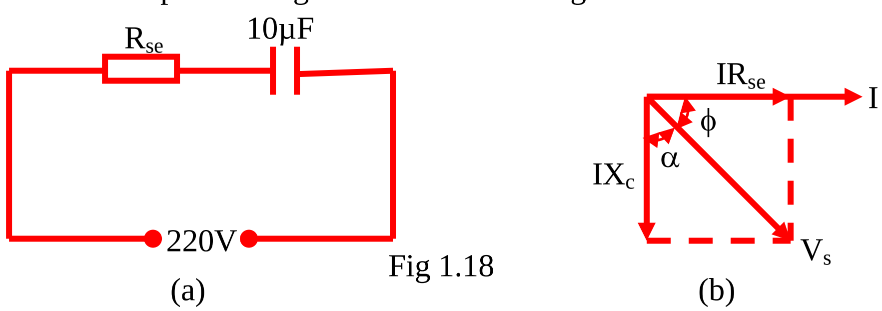


Fig 1.18

In practice, a capacitor has an equivalent resistance (R_{se}) either in series or in parallel with it [fig 1.18(a)]

From the phasor diagram [fig 1.18(b)],

ϕ = actual angle

α = loss angle

and loss factor = $\tan\alpha$

$\therefore \phi + \alpha = 90^0, \Rightarrow \alpha = 90^0 - \phi = 90^0 - 80^0 = 10^0$

and $\tan\alpha = \tan 10^0 = 0.1763$

1.16 BANDWIDTH

When the current in a series RLC circuit is plotted as a function of ω (or f) we obtain the curve as shown in fig 1.19. We notice that the points where the current is 0.707 of the maximum (as indicated on the graph), the corresponding frequencies are ω_1 and ω_2.

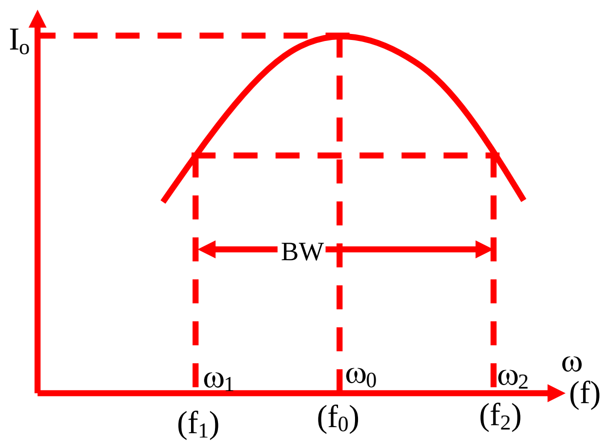


Fig 1.19

The distance between these points ω_1 and ω_2 is known as bandwidth BW. We define the bandwidth, BW, of the resonant circuit to be the difference between the frequencies at which the circuit delivers half of the maximum power. The frequencies ω_1 and ω_2 are called half power frequencies.

1.17 SOLVED PROBLEMS INVOLVING BANDWIDTH AND CIRCUIT Q-FACTOR

Example 1.13

The bandwidth of a series resonance circuit is 130Hz and the resonance frequency is 1300Hz. Find the Q-factor of the circuit

Solution

$$Q_f = \frac{f_0}{BW} = \frac{1300}{130} = 10$$

Example 1.14 obtain the bandwidth in example 1.13 if the Q-factor is reduced by 50%

Solution

$$BW = \frac{f_0}{Q.f}, \text{ where } Q.f = \frac{50}{100} \times 10 = 5$$

$$BW = \frac{1300}{5} = 260Hz$$

At the end of this week, the students are expected to:

- Explain the following terms used in electric networks:
  - ✓ Ideal and practical independent current and voltage sources
  - ✓ Branch
  - ✓ Node
  - ✓ Loop
  - ✓ Network

2.1 TERMS USED IN ELECTRIC NETWORK

The following terms are used in electric network:

(a) Ideal and Practical independent current and voltage sources

(b) Branch

(c) Node

(d) Loop

(e) Network

2.1.1.1 Ideal independent current and voltage sources

Those current and voltage sources, which do not depend on other quantity in the circuit, are called ideal independent current and voltage sources. An ideal independent voltage source is shown in fig. 2.1(a) whereas an ideal independent current source is shown in fig. 2.1(b).

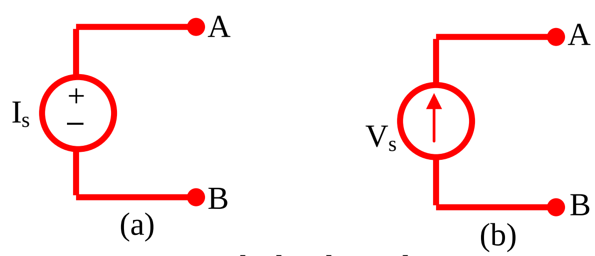


Fig 2.1: Ideal independent sources

2.1.1.2 Practical independent current and voltage sources

A practical independent current source exhibits an internal resistance in parallel with the ideal independent current source. A practical independent voltage source exhibits an internal resistance in series with the ideal independent voltage source.

The schematic representation of a practical independent voltage and current source is shown in fig. 2.2 (a and b) respectively.

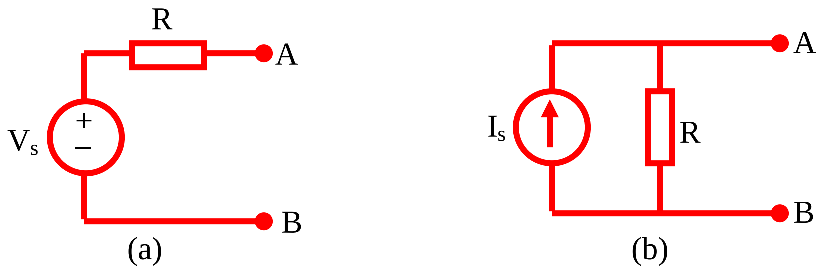


Fig 2.2: practical independent sources

2.1.2 Branch

A branch is part of a network which lies between two junctions. The circuit of fig. 2.3 has three branches.

2.1.3 Node

A node is a junction in a circuit where two or more circuit elements are connected together. The circuit of fig. 2.3 has two nodes.

2.1.4 Loop

A loop is any closed path in a circuit. For example, the circuit of fig. 2.3 has three loops: abefa, bedcb and acdfa

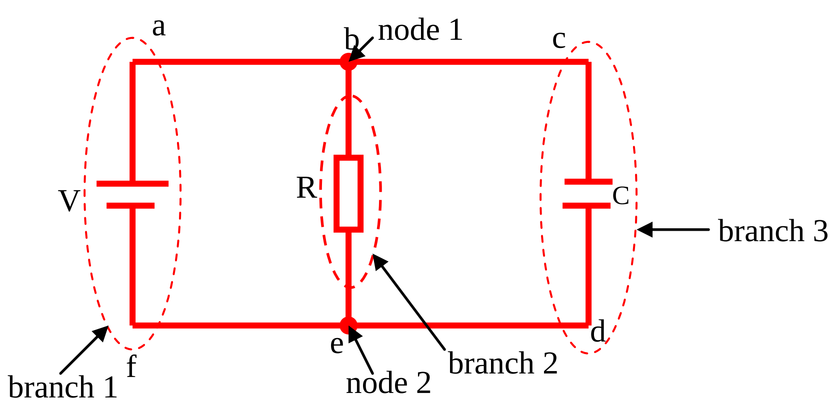


Fig 2.3

At the end of this week, the students are expected to:

- Explain the basic principle of mesh circuit analysis
- Solve problems on mesh circuit analysis

2.1.5 Network

A combination of various electric elements, connected in any manner whatsoever, is called a network. This is shown in fig. 2.3

2.1 BASIC PRINCIPLES OF MESH CIRCUIT ANALYSIS

Consider the network of fig. 2.4, which contains resistances and independent voltage sources and has two meshes. Let the two mesh currents be designated as I_1 and I_2 and all the two may be assumed to flow in the clockwise direction for obtaining symmetry in mesh equations.

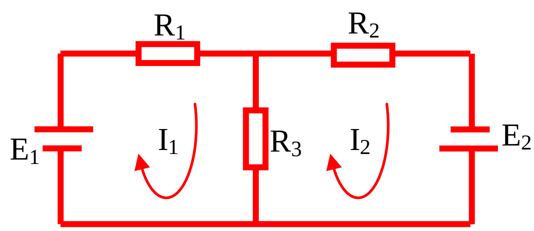


Fig 2.4

Applying KVL to mesh (i), we have

$E_1 = I_1R_1 + I_1R_3 - I_2R_3$

Or

$$E_1 = (R_1 + R_3)\, I_1 - R_3\, I_2 \qquad (2.1)$$

Similarly, from mesh (ii), we have

$E_2 = I_2R_2 + I_2R_3 - I_1R_3$

Or

$$E_2 = -R_3\, I_1 + (R_2 + R_3)\, I_2 \qquad (2.2)$$

The matrix equivalent of the two equations is

$$\begin{pmatrix} (R_1 + R_3) & -R_3 \\ -R_3 & (R_2 + R_3) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

It would be seen that the first item in the first two row i.e (R1 + R3) represents the self resistance of mesh (i) which equals the sum of all resistance in mesh (i). Similarly, the second item

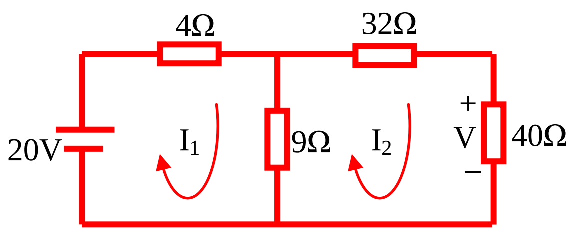
in the first row represents the mutual resistance between meshes (i) and (ii) i.e the sum of the resistances common to mesh (i) and (ii).

The sign of the e.m.f's, while going along the current, if we pass from negative to the positive terminal of a battery, then, its e.m.f is taken positive. If it is the other way around, then battery e.m.f is taken negative.

In the end, it may be pointed out that the directions of mesh currents can be selected arbitrarily.

2.2 SOLVED PROBLEMS ON MESH CIRCUIT ANALYSIS

Example 2.1 Find V in figure 2.5 using mesh analysis



Solution

Fig 2.5

Using KVL for loops 1and 2, we get

$20 = 4\ I_1 + 9(I_1 - I_2)$

Or

$20 = 13\ I_1 - 9I_2$ (2.3)

and $0 = 9(I_2 - I_1) + 32I_2 + 40I_2$

Or

$0 = -9I_1 + 81I_2$ (2.4)

From equations (2.3) and (2.4), we get

$$\begin{pmatrix} 13 & -9 \\ -9 & 81 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \end{pmatrix}$$

$$\Delta_0 = \begin{vmatrix} 13 & -9 \\ -9 & 81 \end{vmatrix} = 1053\text{-}81 = 972$$

$$\Delta\ I_1 = \begin{vmatrix} 20 & -9 \\ 0 & 81 \end{vmatrix} = 1620\text{A}$$

$$\Delta I_2 = \begin{vmatrix} 13 & 20 \\ -9 & 0 \end{vmatrix} = 18OA$$

$I_1 = \Delta I_1/\Delta_0 = 1620/972 = 1.67A$, $I_2 = \Delta I_2/\Delta_0 = 180/972 = 0.185A$

$V = 40I_2 = 40 \times 0.19 \cong 7.4V$

EXAMPLE 2.2 Determine the current in the 4Ω resistor in the circuit shown in figure 2.6 using mesh analysis

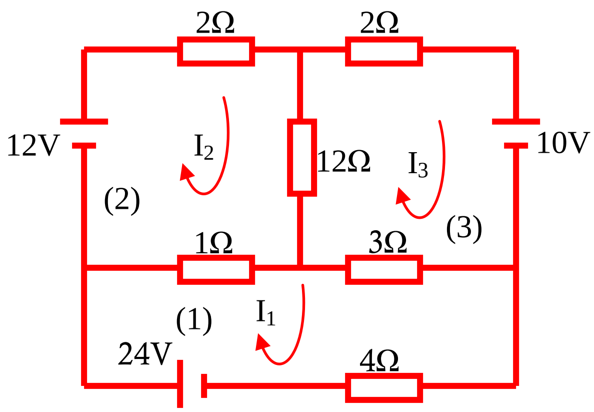


Fig 2.6

Solution

For mesh 1, we can write

$24 = 1(I_1 - I_2) + 3(I_1 - I_3) + 4I_1$

Or $24 = 8I_1 - I_2 - 3I_3$ (2.5)

For mesh 2,

$12 = 2I_1 + 12(I_2 - I_3) + 1(I_2 - I_1)$

Or $-12 = I_1 - 15I_2 + 12I_3$ (2.6)

For mesh 3,

$-10 = 12(I_3 - I_2) + 2I_3 + 3(I_3 - I_1)$

Or $10 = 3I_1 + 12I_2 - 17I_3$ (2.7)

From eqtn. (2.5),(2.6) and (2.7), we get

$$\begin{pmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{pmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ -12 \\ 10 \end{bmatrix}$$

$$\therefore \Delta_0 = \begin{vmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{vmatrix} = 8 \begin{vmatrix} -15 & 12 \\ 12 & -17 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 12 \\ 3 & -17 \end{vmatrix} - 3\begin{vmatrix} 1 & -15 \\ 3 & 12 \end{vmatrix}$$

$= 888 – 53 – 171 = 664$

$$\therefore \Delta_1 = \begin{vmatrix} 24 & -1 & -3 \\ -12 & -15 & 12 \\ 10 & 12 & -17 \end{vmatrix} = 24 \begin{vmatrix} -15 & 12 \\ 12 & -17 \end{vmatrix} - (-1)\begin{vmatrix} -12 & 12 \\ 10 & -17 \end{vmatrix} -3\begin{vmatrix} -12 & -15 \\ 10 & 12 \end{vmatrix}$$

$= 2664 + 84 – 18 = 2730$

$I_1 = \Delta_1/\Delta_0 = 2730/664 = 4.111A$

$\therefore$ $I_1 = 4.111A$, is the current in the 4Ω resistor

At the end of this week, the students are expected to:

- ♦ Explain the basic principle of nodal analysis
- ♦ Solve problems on nodal analysis

2.4 BASIC PRINCIPLE OF NODAL ANALYSIS

Consider the circuit of fig 2.7 which has three nodes. One of these i.e node 3 has been taken in as the reference node. V_A represents the potential of node 1 with reference to the reference node (or zero potential node). Similarly, V_B is the potential difference between node 2 and node 3. Let the current directions which have been chosen arbitrary be as shown

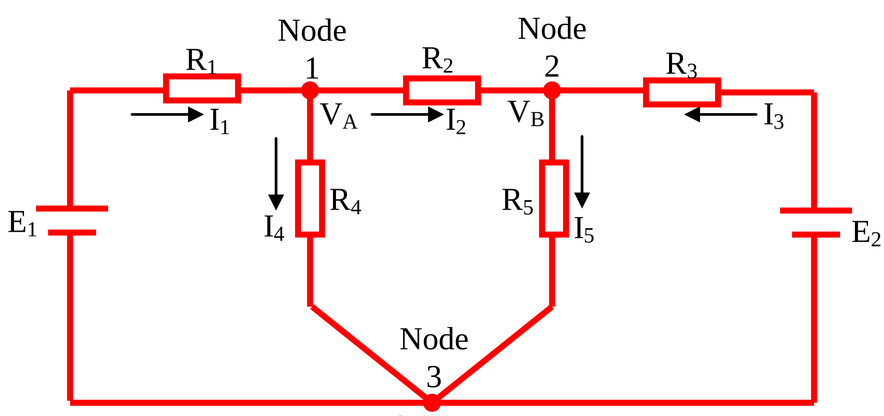


Fig 2.7

For node 1, the following current equation can be written with the help of KCL.

$$I_1 = I_4 + I_2 \qquad (2.8)$$

Now $E_1 = I_1R_1 + V_A \therefore I_1 = (E_1 - V_A)/R_1$

Obviously $I_4 = V_A/R_4$, Also $V_A = I_2R_2 + V_B$

$$\therefore I_2 = (V_A - V_B)/R_2$$

Substituting these values in eqtn (2.8) above, we get,

$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2}$$

Simplifying the above, we have

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{V_B}{R_2} = \frac{E_1}{R_1} \qquad (2.9)$$

The current equation for node 2 is $I_5 = I_2 + I_3$ (2.10)

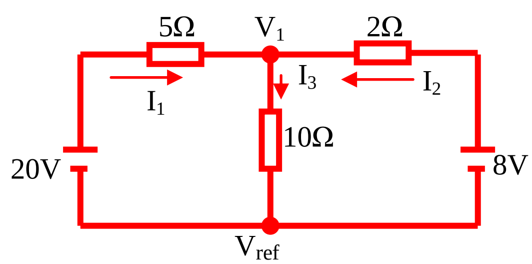
$$\text{Or } \frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3}$$

$$\text{Or } \frac{-V_A}{R_2} + V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) = \frac{E_2}{R_3} \qquad (2.11)$$

The node voltages in equation (2.9) and (2.11) are the unknowns and when determined by a suitable method, result in the network solution. After finding different node voltages, various current can be calculated by using ohm's law.

2.5 SOLVED PROBLEMS ON NODAL ANALYSIS

Example 2.3 obtain the node voltage and the branch current in the circuit shown in fig 2.8



Solution

Fig 2.8

For node 1, applying KCL gives

$I_3 = I_1 + I_2$

$$\text{Or } \frac{V_1}{10} = \frac{20 - V_1}{5} + \frac{8 - V_1}{2}$$

$$\Rightarrow \frac{V_1}{10} + \frac{V_1}{2} + \frac{V_1}{5} = \frac{20}{5} + \frac{8}{2} = 8$$

Or $V_1 + 5V_1 + 2V_1 = 8 \text{ x } 10 = 80$

$8V_1 = 80$

$\therefore V_1 = {}^{80}/_{8} = 10V$

$$I_1 = \frac{20 - 10}{5} = 2A,\ I_2 = \frac{8 - 10}{2} = -1A$$

$I_3 = {}^{10}/_{10} = 1A$

Example 2.4 Determine the voltages at nodes b and c in the network shown in figure 2.9 using nodal analysis

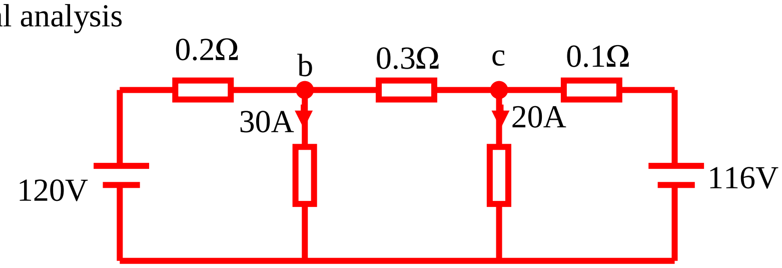


Fig 2.9

Solution:

Let the voltages at nodes b and c be V_b and V_c respectively.

Using nodal analysis for node b, we get

$$\frac{V_b - 120}{0.2} + \frac{V_b - V_c}{0.3} + 30 = 0 \text{ or } 5V_b - 2V_c = 342 \qquad (2.12)$$

Using nodal analysis for node c, we get

$$\frac{V_c - V_b}{0.3} + \frac{V_c - 116}{0.1} + 20 \text{ or } -V_b + 4V_c = 342 \qquad (2.13)$$

From equation (2.13), $V_b = 4V_C - 342$ (2.14)

From equation. (2.12) and (2.14), we get

$$5(4V_c - 342) - 2V_c = 342 \text{ or } V_c = 114V$$

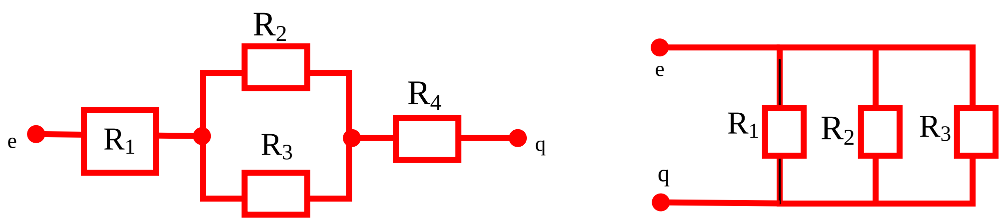
$$\therefore V_b = 4V_c - 342 = 4 \text{ x } 114 - 342 = 114V$$

At the end of this week, the students are expected to:

- ♦ Reduce a complex network to its equivalent
- ♦ Identify star and delta networks

3.1 REDUCTION OF A COMPLEX NETWORK TO ITS SERIES OR PARALLEL EQUIVALENT

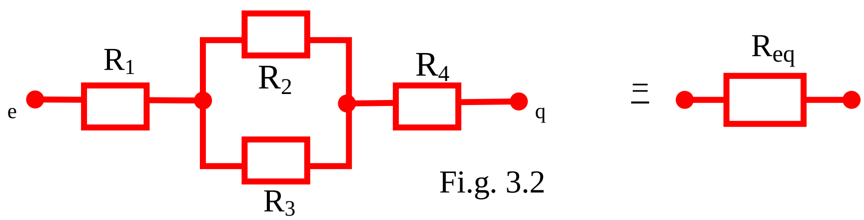
The network of fig. 3.1 (a) and (b) can be reduced to its equivalent as shown below.



(a) Fig. 3.1 (b)

Consider fig. 3.1(a), the equivalent resistance of the network is shown in fig. 3.2

Req = $R_1 + R_4 + (R_2 // R_3)$



Fi.g. 3.2

The equivalent resistance of fig. 3.1(b), is shown in fig. 3.3

$1/Req = 1/R_1 + 1/R_2 + 1/R_3$

$\Rightarrow Req = R_1R_2R_3 / (R_1R_2 + R_1R_3 + R_2R_3)$

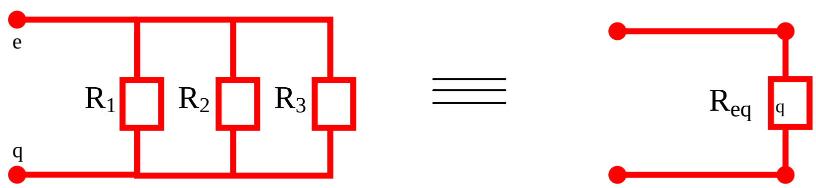


Fig 3.3

Also the circuit of fig 3.1(c) can be reduced to its equivalent as under

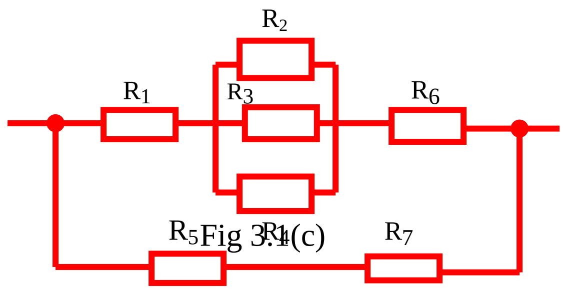


Fig 3.1(c)

$R_{p1} = R_2R_3R_4 / (R_2R_3 + R_2R_4 + R_3R_4)$

The circuit is reduced to the one shown in fig 3.1(d)

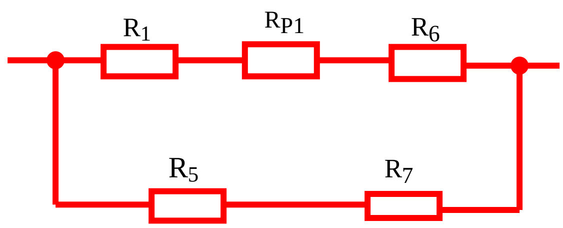


Fig 3.1(d)

From fig 3.1(d),

$R_{S1} = R_1 + R_{P1} + R_6$

The circuit becomes as shown in fig 3.1(e)

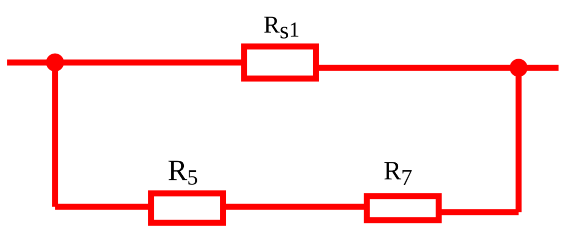


Fig 3.1(e)

$R_{S2} = R_5 + R_7$

The circuit becomes as shown in fig 3.1(f)

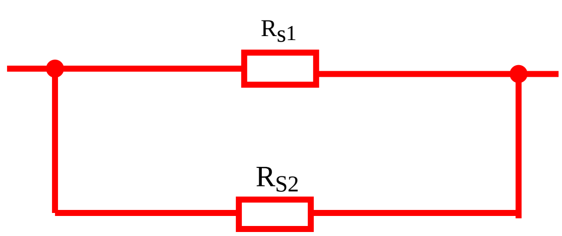


Fig 3.1(f)

$Req = R_{S2} + R_{S1}$

The circuit is finally reduced as shown in fig 3.1(g)

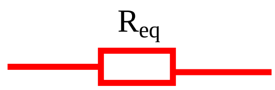


Fig 3.1(g)

3.2 IDENTIFICTION OF STAR AND DELTA NETWORKS

Delta (△) and Star (Y) networks are identified for being having three terminals. Thus, they are called three-terminal equivalent networks. Figure 3.4 shows a delta-network whereas a star-network is depicted in figure 3.5

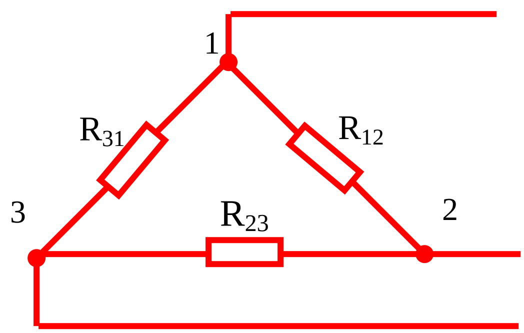


Fig. 3.4 Delta-network

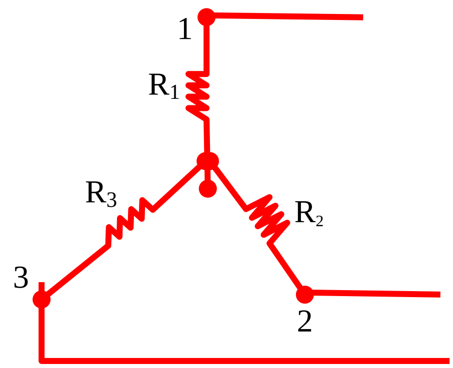


Fig. 3.5 Star-network

At the end of this week, the students are expected to:

- Derive the formula for the transformation of a delta to a star network and vice versa
- Solve problems on Delta to star transformation

3.3 DERIVATION OF FORMULAE FOR THE TRANSFORMATION OF A DELTA TO A STAR NETWORK AND VICE VERSA

3.3.1 Derivation of formulae for the transformation of a delta to a star network.

The circuit of fig. 3.4 and 3.5 can be redrawn as shown in figure 3.6 and 3.7 respectively.

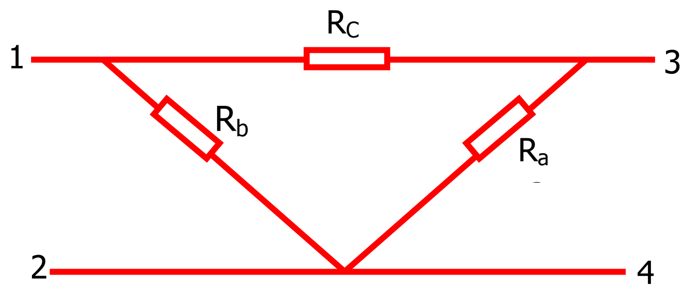


Fig 3.6

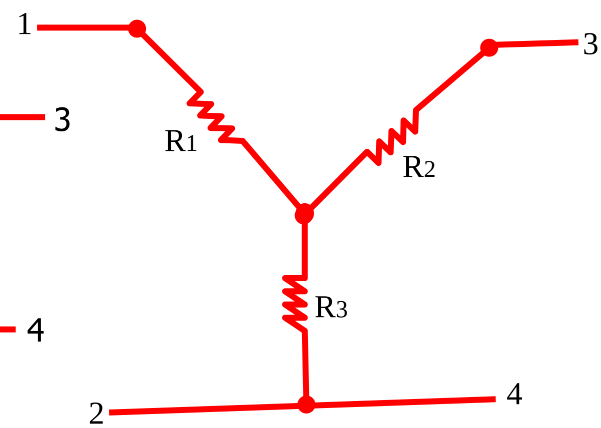


Fig 3.7

Consider figure 3.6 and 3.7, the resistance between terminal 1 and 2 is

$$R_{12}\ (Y) = R_1 + R_3 \qquad (3.1)$$

$$R_{12}(\triangle) = Rb\ //\ (Ra+Rc) \qquad (3.2)$$

Setting $R_{12}(Y) = R_{12}(\triangle)$ gives

$$R_{12} = R_1 + R_3 = \frac{Rb(Ra+Rc)}{Ra + Rb + Rc} \qquad (3.3)$$

$$\text{Similarly, } R_{13} = R_1 + R_2 = \frac{Rc(Ra+Rb)}{Ra + Rb + Rc} \qquad (3.4)$$

$$\text{and } R_{34} = R_2 + R_3 = \frac{Ra(Rb+Rc)}{Ra + Rb + Rc} \qquad (3.5)$$

Subtracting eqtn. (3.5) from (3.3), we get

$$R_1 - R_2 = \frac{R_C(R_b - R_a)}{R_a + R_b + R_c} \qquad (3.6)$$

Adding eqtn. (3.4) and (3.6) gives

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c} \qquad (3.7)$$

Subtracting eqtn. (3.6) from (3.4) gives

$$R_2 = \frac{R_cR_a}{R_a+R_b+R_c} \qquad (3.8)$$

Subtracting eqtn. (3.7) from eqtn. (3.3) gives

$$R_3 = \frac{R_aR_b}{R_a+R_b+R_c} \qquad (3.9)$$

∴ Eqtn. (3.7) to (3.9) is the required conversion formulae

3.3.2 Derivation of formulae for the transformation of a star to a delta network

Multiplying equation (3.7) and (3.8), (3.8) and (3.9), (3.7) and (3.9) and adding them together gives,

$$R_1R_2 + R_2R_3 + R_1R_3 = \frac{R_aR_bR_c\,(R_aR_bR_c)}{(R_a+R_b+R_c)^2}$$

$$= \frac{R_aR_bR_c}{R_a+R_b+R_c} \qquad (3.10)$$

Dividing eqtn. (3.10) by each of eqtn. (3.7) to (3.9) gives

$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1} \qquad (3.11)$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2} \qquad (3.12)$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3} \qquad (3.13)$$

Equation (3.11) to (3.13) is the required conversion formulae

3.4 SOLVED PROBLEMS ON DELTA AND STAR TRANSFORMATION

Example 3.1: Transform the network of fig. 3.8 to an equivalent star network.

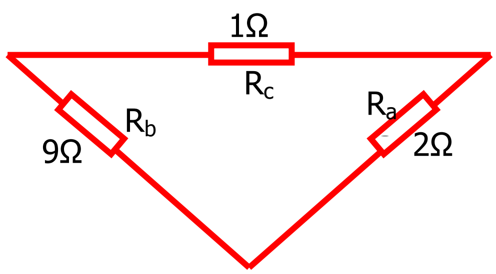


Fig 3.8

Solution

$$R_1 = \frac{R_a R_b}{Ra + Rb + Rc} = \frac{1 \text{ X } 9}{2+9+1} = 0.75\Omega$$

$$R_2 = \frac{R_a R_c}{Ra + Rb + Rc} = \frac{2\text{X}1}{2+9+1} = \frac{2}{12} = 1/6\Omega$$

$$R_3 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{9\text{X}2}{9+2+1} = 1.5\Omega$$

The equivalent star network is shown in fig 3.9

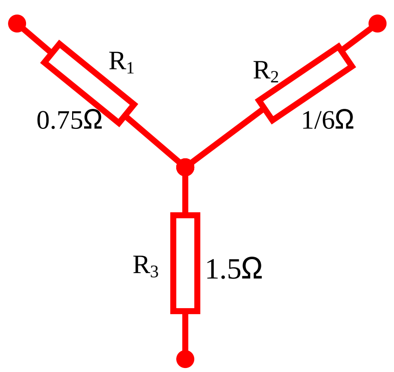


Fig. 3.9

Example 3.2 Transform the network of fig 3.9 to an equivalent delta network

Solution

$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$\underline{0.75} \quad \frac{\text{x } {}^1/_6 + {}^1/_6 \text{ x } 1.5 + 1.5 \text{ x } 0.75}{0.75} = 2\Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$$

$$\frac{0.75 \text{ x } {}^1/_6 + {}^1/_6 \text{ x } 1.5 + 1.5 \text{ x } 0.75}{{}^1/_6} = 9\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$$

$$\frac{0.75 \text{ x } {}^1/_6 + {}^1/_6 \text{ x } 1.5 + 1.5 \text{ x } 0.75}{1.5} = 1\Omega$$

At the end of this week, the students are expected to:

- ♦ Explain the meaning of duality principle

Establish duality between resistance, conductance, inductance, capacitance, voltage and current.

3.5 DUALITY PRINCIPLE

Consider for example, the relationship between series and parallel circuits. In a series circuit, individual voltages are added and in a parallel circuit, individual currents are added. It is seen that while comparing series and parallel circuits,voltage takes the place of current and current takes the place of voltage. Such a pattern is known as duality principle.

3.6 DUALITY BETWEEN RESISTANCE, CONDUCTANCE, INDUCTANCE, CAPACITANCE, VOLTAGE AND CURRENT.

The duality between resistance, conductance, inductance, capacitance, voltage and current is tabulated as shown in table 3.1

Table 3.1

	Series Circuits	Parallel circuits
(a)	$I_1 = I_2 = I_3$ ------------In	$V_1 = V_2 = V_3$ ---------Vn
(b)	$V_T = V_1 + V_2 + V_3 + ---+Vn$	$I_T = I_1 + I_2 + I_3 + ---- + In$
(c)	$R_T = R_1 + R_2 + R_3 + ---+ Rn$	$G_T=G_1+G_2+ G_3 + ---+ Gn$
(d)	$^1/G_T=^1/G_1+^1/G_2+^1/G_3+--+^1/Gn$	$^1/R_T=^1/R_1+^1/R_2+^1/R_3+-+ ^1/Rn$
(e)	$L_T = L_1 + L_2 + L_3 + ---+ Ln$	$C_T=C_1+C_2+C_3+------+ Cn$
(f)	$^1/C_T=^1/C_1+^1/C_2+^1/C_3+---+^1/Cn$	$^1/L_T=^1/L_1+^1/L_2+^1/L_3+----+^1/Ln$

At the end of this week, the students are expected to:

- Find the duality of a network
- Solve network problems using duality principle

3.7 DUALITY OF A NETWORK

If two electric networks are governed by the same type of equations, these two networks are known as dual networks.

Consider the circuit shown in fig. 3.12. The voltage across the two resistors is given by

$$V=I(R_1+R_2) \quad (3.14)$$

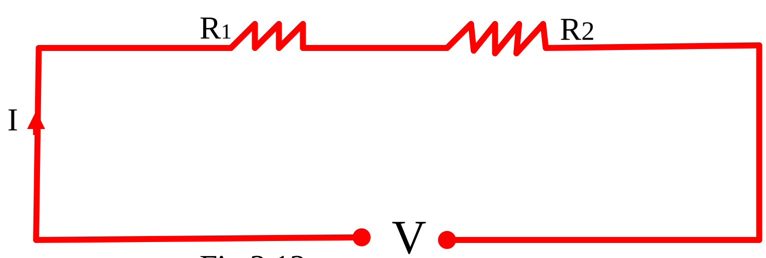


Fig 3.12

The dual of the circuit elements in eqtn. (3.14) are as follows

V = I

I = V

R1 + R2 = G1 + G2

Therefore the dual of equation (3.14) is

$$I = V(G_1 + G_2) \quad (3.15)$$

Also, the dual of series circuit is parallel. Hence, the dual circuit of fig. 3.12 is redrawn as shown in fig. 3.13

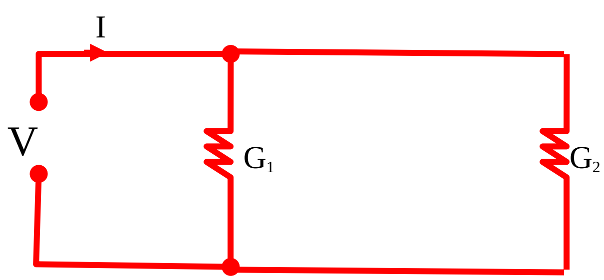
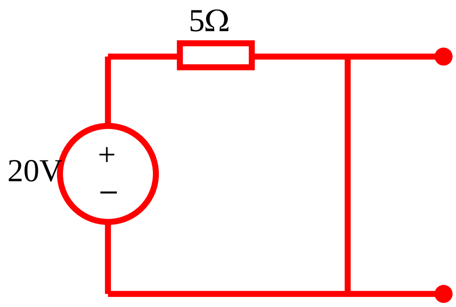


Fig. 3.13

3.8 SOLVED NETWORK PROBLEMS USING DUALITY PRINCIPLE

Example 3.3 Draw the dual network of the one shown in fig. 3.14



Solution

Step 1: The dual of 20V voltage source is 20A current source, as shown in fig. 3.15(a)

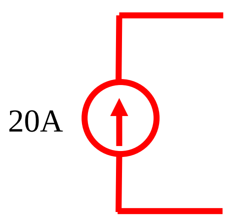


Fig. 3.15(a)

Step 2: The dual of 5Ω resistor is 1/5 series = 1/5Ω resistor in parallel

Step 3: Since the 20V voltage source is in series with 5Ω resistor, it dual counterpart is 20A current source in parallel with 1/5Ω. This is shown in fig. 3.15(b)

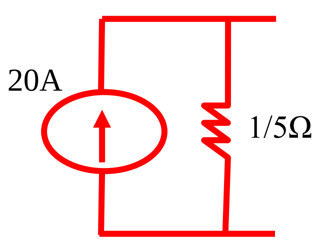


Fig. 3.15(b):- dual network of fig. 3.14

Example 3.4 Draw the dual network of fig 3.16

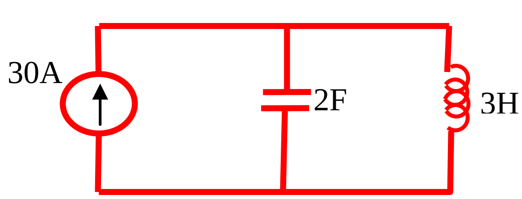


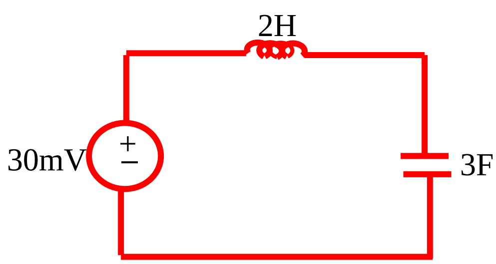
Fig. 3.16

Solution

Since the current source, capacitor and inductor are all connected in parallel, their dual counterparts will be connected in series.

- ❖ The dual of 30A current source is 30V voltage source
- ❖ The dual of 2F capacitor is 2H inductor
- ❖ The dual of 3H inductor is 3F capacitor

The dual network of Fig. 3.16 is shown in Fig. 3.17



At the end of this week, the students are expected to:

- ♦ State Thevenin's theorem
- ♦ Explain the basic principle of Thevenin's theorem
- ♦ Solve problems on simple network using Thevenin's theorem

4.1 THEVENIN'S THEOREM

Thevenin's theorems state that a linear two terminal network may be reduced to an equivalent circuit consisting of a single voltage source in series with a single resistor as shown in figure 4.1

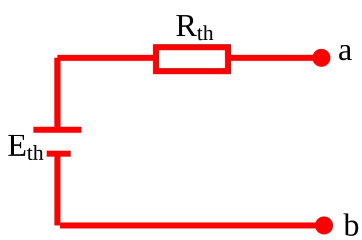


Fig 4.1: Thevenin's equivalent circuit

4.2 BASIC PRINCIPLES OF THEVENIN'S THEOREM

The following principles provide a technique that converts any circuit into its Thevenin's equivalent:

1. Remove the load from the circuit
2. Label the resulting two terminals. We will label them as a and b, although any notation may be used
3. Set all sources in the circuit to zero

   Voltage source are set to zero by replacing them with short circuits.

   Current sources are set to zero by replacing them with open circuits.
4. Determine the Thevenin's equivalent resistance, R_{th}, by calculating the resistance "seen" between terminals a and b. It may be necessary to redraw the circuit to simplify this step.
5. Replace the sources removed in (3), and determine the open circuit voltage between the terminals. The resulting open circuit voltage will be the value of the Thevenin's voltage, E_{th}.
6. Draw the Thevenin's equivalent circuit using the resistance determined in (4) and the voltage calculated in (5). As part of the resulting circuit, include that portion of the network removed in (1)

4.3 SOLVED PROBLEMS ON SIMPLE NETWORK USING THEVENIN'S THEOREM

Example 4.1 Obtain the Thevenin's equivalent circuit for the active network in fig 4.2(a).

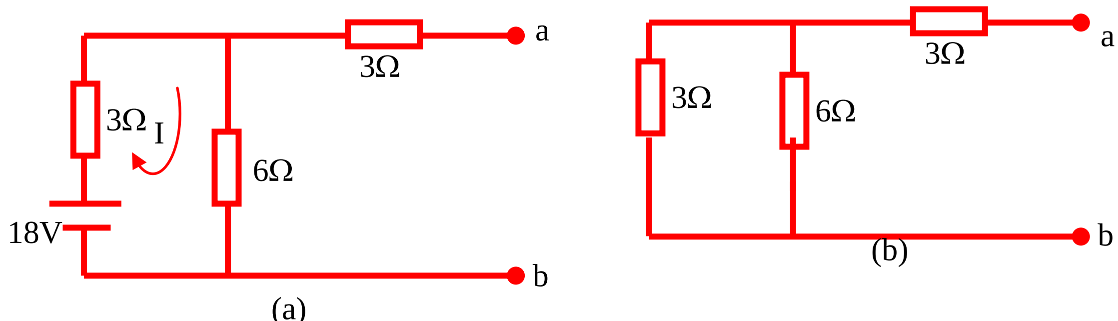


Fig 4.2

Solution

With terminals ab open, the current through the 3Ω and 6Ω resistor is

$$I = \frac{18}{3+6} = 2A$$

The Thevenin's voltage V_{Th}, is the voltage across terminal a-b.

Hence, $V_{ab} = V_{th} = I \times 6\Omega = 2 \times 6 = 12V$

The Thevenin's resistance can be obtained by shorting out the 18V sources [fig 4.2(b)] and finding the equivalent resistance of this network at terminals ab:

$$R_{Th} = 3 + \frac{3 \times 6}{3+6} = 5\Omega$$

The Thevenin's equivalent circuit is shown in fig 4.3

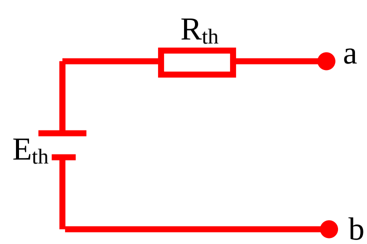


Fig 4.3: Thevenin's equivalent circuit

Example 4.2 Applying Thevenin's theorem to find the current through the resistance R as shown in figure 4.4(a)

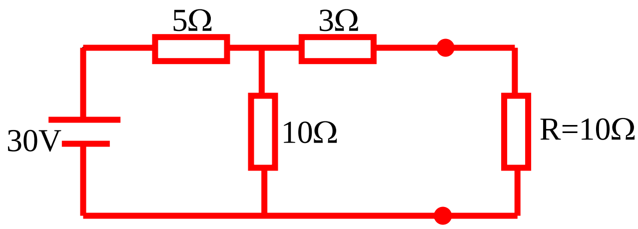


Fig 4.4(a)

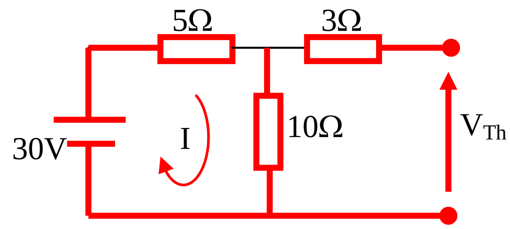


Fig 4.4(b)

Solution

With R disconnected [fig 4.4(b)], the current in the loop is

$$I = \frac{30}{5 + 10} = 2A$$

$$V_{Th} = V_{ab} = 10 \text{ x } I = 10 \text{ x } 2 = 20V$$

To find R_{Th} we short circuit the source as shown in fig 4.4(c)

$$R_{th} = 3 + \frac{5 \text{ x } 10}{5 + 10} = {}^{19}/_{3}\Omega$$

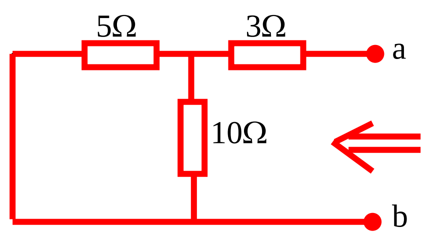


Fig 4.4(c)

To find the current through R, we replace the left side of ab by its Thevenin's equivalent and connect the load resistance as shown in fig 4.4(d)

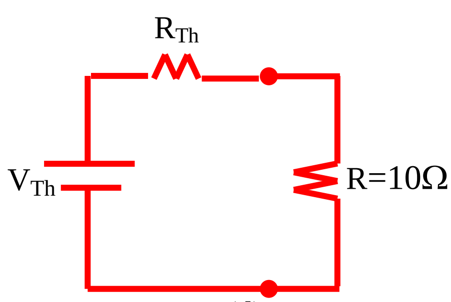


Fig 4.4(d)

$$I = \frac{V_{Th}}{R_{Th} + R} = \frac{20}{{}^{19}/_{3} + 10} = 1.224A$$

At the end of this week, the students are expected to:

- ♦ State Norton's theorem
- ♦ Explain the basic principle of Norton's theorem
- ♦ Compare Norton's theorem with Thevenin's theorem
- ♦ Solve simple problems on Norton's theorem

4.5 NORTON'S THEOREM

Norton's theorem state that a linear two terminal network may be reduced to an equivalent circuit consisting of a single current source and a single shunt resistor as shown in fig 4.8

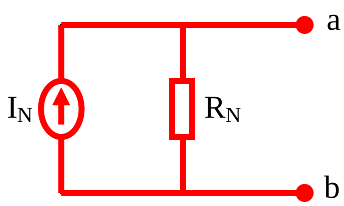


Fig 4.8: Norton's equivalent circuit

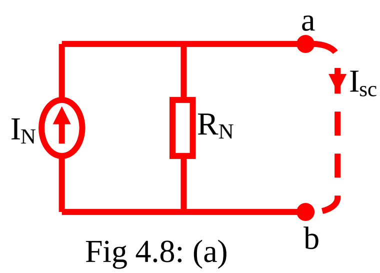


Fig 4.8: (a)

4.6 BASIC PRINCIPLE OF NORTON'S THEOREM

As seen from fig 4.8(a), to find the Norton's current I_N, we determine the short circuit current flowing from terminal a and b. It is evident that the short circuit current i_{sc} in fig 4.8(a) is I_N (Norton's current).

Thus $I_N = i_{sc}$ (4.1)

We find Norton's resistance (R_N) in the sane way we find Thevenin's resistance (R_{Th}). Thus

$R_N = R_{Th}$ (4.2)

Finally, after obtaining I_N and R_N we draw the Norton's equivalent circuit as shown in fig 4.8

4.7 COMPARISM OF NORTON'S THEOREM WITH THEVENIN'S THEOREM

Norton's theorem is related to Thevenin's theorem as follows:

$R_N = R_{Th}$ as in eqtn. (4.2)

$$\therefore I_N = V_{Th}/R_{Th} \qquad (4.3)$$

Also, $I_N = V_{Th}/R_N$ (4.4)

4.8 SOLVED PROBLEMS USING NORTON'S THEOREM

Example 4.5 Determine the Norton's equivalent circuit of fig 4.9

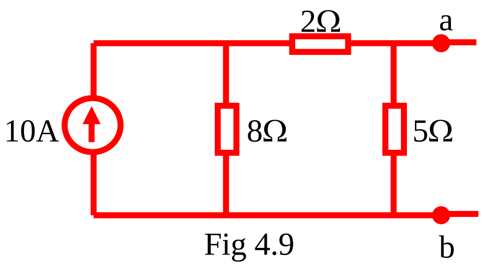
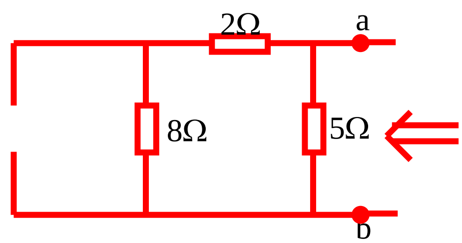


Fig 4.9

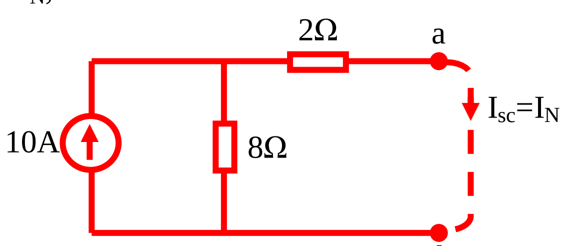


Solution

To find R_N, we open circuit the 10A current source as shown in fig 4.10(a)

$$\therefore R_N = \frac{(8+2) \times 5}{8+2+5} = \frac{50}{15} = {}^{10}/_{3}\Omega$$

To find I_N, we short circuit terminal ab as shown in fig 4.10(b).



$$I_N = \frac{10 \times 8}{8+2} = 8A$$

Fig 4.10(b)

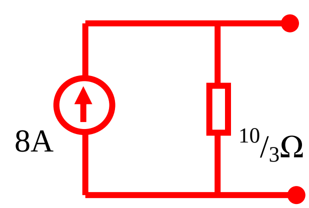


Fig 4.10(c)

The Norton's equivalent circuit is shown in fig 4.10(c)

Example 4.6 Determine the current I_L in the circuit shown in figure 4.11(a) by using the Norton's theorem

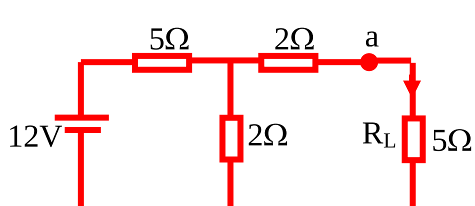


Fig 4.11(a)

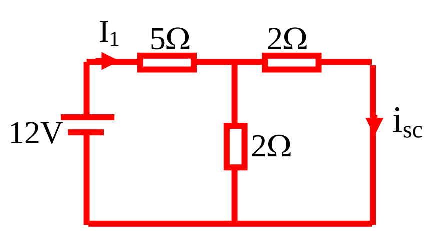


Fig 4.11(b)

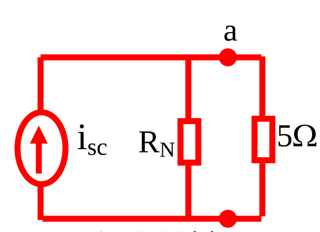


Fig 4.11(c)

Solution

With R_L disconnected and replaced by short circuit [fig 4.11(b)],

$$I_1 = \frac{12}{5 + \left(\frac{2 \times 2}{2 + 2}\right)} = 2A$$

$$\therefore\ i_{sc} = \frac{2 \times 2}{2 + 2} = 1A = I_N$$

To find R_N, the voltage is replaced by a short circuit,

$$\therefore R_N = 2 + \frac{5 \times 2}{5 + 2} = \frac{24}{7}\Omega$$

The Norton's equivalent circuit with R_L connected across terminal ab is shown in fig 4.11(c).

$$\therefore I_L = \frac{i_{sc}\ R_N}{R_N + 5} = \frac{1 \times {}^{24}/_{7}}{{}^{24}/_{7} + 5} = 0.407A$$

At the end of this week, the students are expected to:

- ♦ State Millman's theorem
- ♦ Explain the basic principle of Millman's theorem
- ♦ Solve network problems using Millman's theorem

4.9 MILLIMAN'S THEOREM

This states that any number of parallel voltage sources E_1, E_2,……,E_n having internal resistance R_1, R_2,…….,R_n respectively can be replaced by a single equivalent source E in series with an equivalent series resistance R.

4.10 BASIC PRINCIPLE OF MILLIMAN'S THEOREM

In circuits of the type shown in figure 4.12, the voltage sources may be replaced with a single equivalent source as shown in figure 4.13

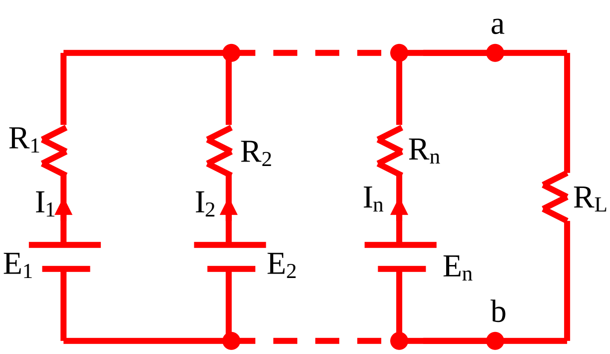


Fig 4.12

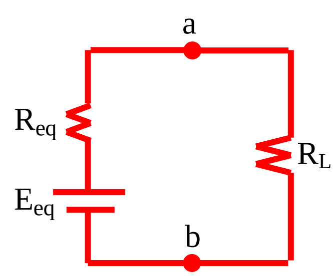


Fig 4.13

The values of I_1, I_2 ……I_n would be determined using ohm's law:

$$I_1 = E_1/R_1,\ \ I_2 = E_2/R_2,\ \ldots\ldots\ I_n = E_n/R_n$$

Hence the equivalent current in the circuit is given by

$$I_{eq} = I_1 + I_2 + \ldots.. + I_n \qquad (4.5)$$

The equivalent resistance R_{eq} is obtain by short circuiting the voltage sources. The resistance seen via terminal ab when R_L is removed is given by

$$R_{eq} = R_1 // R_2 // \ldots. // R_n \qquad (4.6)$$

Which may be determined as

$$R_{eq} = \frac{1}{{}^1/R_1 + {}^1/R_2 + \ldots. + {}^1/R_n} \qquad (4.7)$$

The general expression for the equivalent voltage is

$$E_{eq} = I_{eq}R_{eq} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3} + \ldots.. + \frac{E_n}{R_n}}{{}^1/R_1 + {}^1/R_2 + {}^1/R_3 + \ldots..+ {}^1/R_n} \qquad (4.8)$$

4.11 SOLVED NETWORK PROBLEMS USING MILLIMAN'S THEOREM

Example 4.7 Using Milliman's theorem to find the common voltage across terminals a and b in the circuit of fig 4.14

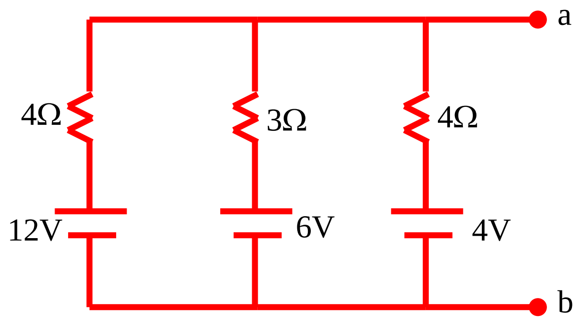


Fig 4.14

Solution

$$V_{ab} = V_{eq} = \frac{\frac{12}{4} + \frac{6}{3} + \frac{4}{4}}{\frac{1}{4} + \frac{1}{3} + \frac{1}{1}} = 3.79\text{v}$$

Example 4.8 Using the Milliman's theorem, determine the current through R_L in the circuit shown in fig 4.15(a)

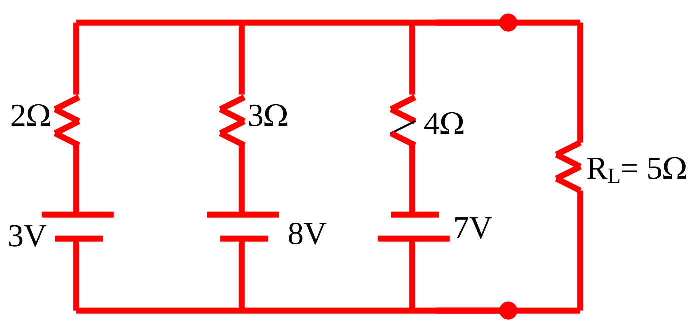


Fig 4.15(a)

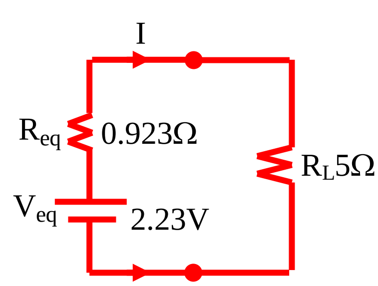


Fig 4.15(b)

Solution

$$V_{eq} = \frac{\frac{3}{2} + \frac{8}{3} - \frac{7}{4}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = 2.23\text{V}$$

$$R_{eq} = \frac{1}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = 0.923\Omega$$

The equivalent circuit is shown in fig 4.15(b)

$$\therefore I = \frac{V_{eq}}{R_{eq} + R_L} = \frac{2.23}{0.923 + 5} = 0.3765A$$

At the end of this week, the students are expected to:

- ♦ State Reciprocity theorem
- ♦ Explain the basic principle of Reciprocity theorem
- ♦ Solve network problems using Reciprocity theorem

4.12 RECIPROCITY THEOREM

This state that in any linear bilateral network , if a source of e.m.f E in any branch produces a current I in any other branch, then the same e.m.f E acting in the second branch would be the same current I in the first branch.

4.13 BASIC PRINCIPLE OF RECIPROCITY THEOREM

When applying the reciprocity theorem, the following principle must be followed:

1. The voltage sources is replaced by a short circuit in the original location
2. The polarity of the source in the new location is such that the current direction in that branch remains unchanged.

4.14 SOLVED NETWORK PROBLEMS USING RECIPROCIUTY THEOREM

Example 4.9: Consider the circuit of figure 4.16:

(a) Calculate the current I

(b) Remove voltage source E and place it into the branch with R_3. Show that the current through the branch which formerly had E is now the same as the current I

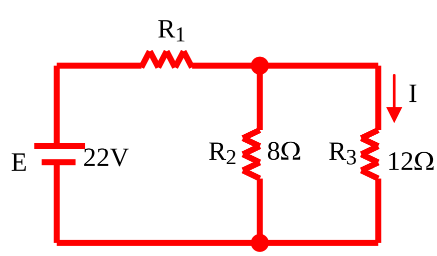


Fig 4.16

Solution

(a) $$V(12\Omega) = \left(\frac{8\Omega//12\Omega}{4\Omega + (8\Omega//12\Omega)}\right)^{(22)} = \frac{4.8}{8.8} \times 12 = 12V$$

$$I = \frac{V(12\Omega)}{12\Omega} = \frac{12}{12} = 1A$$

(b) Removing the voltage sources E and placing it into the branch with R_3 gives the circuit shown in figure 4.17

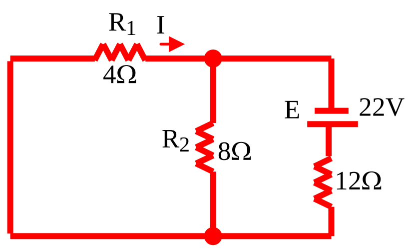


Fig 4.17

$$V(4\Omega) = \left(\frac{4\Omega//8\Omega}{12\Omega + (4\Omega//8\Omega)}\right)^{(22)} = \frac{2.6}{14.6} \text{ x } 22 = 4V$$

$$I = \frac{V(4\Omega)}{4\Omega} = \frac{4}{4} = 1A$$

Hence, current I is the same in both cases

Example 4.10 In the network of fig 4.18(a), find (i) ammeter current when battery is at A and ammeter at B and (ii) when battery is at B and ammeter at point A

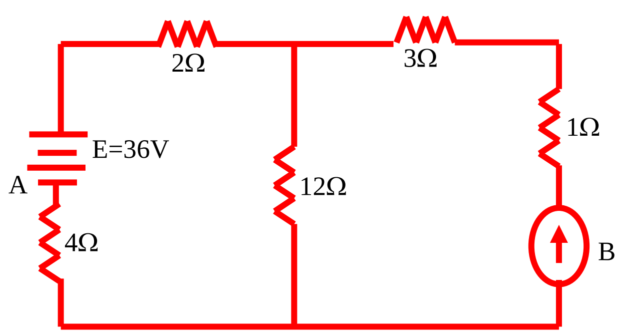


Fig 4.18(a)

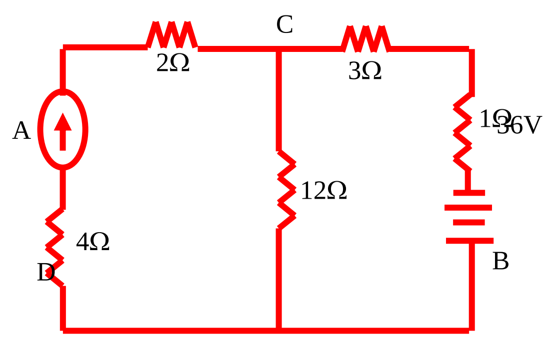


Fig 4.18(b)

Solution

(i) Effective resistance between points C and B [fig4.18 (a)] is

$R_{CB} = 12 \times {}^{4}/_{16} = 4\Omega$

Total circuit resistance = 2 + 3 + 4 = 9Ω

Battery current = ${}^{36}/_{9} = 4A$

Ammeter current = $4 \times {}^{12}/_{16} = 3A$

(ii) Effective resistance between points C and D [fig 4.18(b)] is

$R_{CD} = 12 \times {}^{6}/_{18} = 4\Omega$

Total circuit resistance = 4 + 3 + 1 = 8Ω

Battery current = ${}^{36}/_{8} = 4.5A$

Ammeter current = $4.5 \times {}^{12}/_{18} = 3A$

Hence, ammeter current in both cases is the same.